

# **Integral Modelling of the Dilution and Lift-off of Ground Based Buoyant Plumes and Comparison with Wind Tunnel Data**

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A Deliverable produced under Work Package 7 of the EC URAHFREP Project, Contract No. ENV4-CT97-0630

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## Executive Summary

The occurrence and quantification of the mitigation of anhydrous hydrogen fluoride (AHF) clouds due to buoyancy generation is currently very uncertain. The EC URAHFREP research programme, of which the work presented in this report is a part, is aimed at reducing the uncertainties to enable more reliable estimates to be made of the hazards associated with accidental AHF releases. An important aspect of the uncertainty is the influence of buoyancy on the dilution and lift-off of initially ground based clouds. The work presented here is aimed at developing simple models capable of describing the behaviour of such clouds resulting from continuous ground level sources.

A simple integral plume model for plume lift-off is presented. The model is based on traditional free plume models, modified to account for the reduced area for entrainment due to the presence of the ground. Apart from this modification of entrainment and also introduction of an algorithm to relate the plume centroid to plume concentration maximum, this simple model has no further modifications to account for dynamical effects of the ground on the plume. Comparisons have been made with existing and also new wind tunnel data on buoyancy conserving plumes collected by partners in the URAHFREP programme. These comparisons indicate that the simple model is capable of adequately representing plume dilution and lift-off for plumes arising from sources which are not too wide compared with the conserved buoyancy length scale. For wide sources, the model significantly underpredicts the suppression of plume rise.

Further work has been undertaken to attempt to overcome the shortcomings of the simple model. Parametrisation of the buoyant contribution to dilution for the ground based plume (prior to lift-off) has been investigated. Incorporating such buoyant entrainment in ground based models, such as DRIFT, may enable them to be applied up to the point of lift-off. Data of Hall and Walker ref.[7] indicate possible critical lift-off parameter values which could be used as the transition point from a grounded to an elevated plume model. Improved modelling of the area sources of Hall and Walker ref.[7] is also considered. However, difficulties in the modelling of the subsequent plume lift-off remain with regard to suppression of plume rise from wide source and more work is required in this area.

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# 1 Introduction

Reliable estimation of the dispersion behaviour of anhydrous hydrogen fluoride (AHF) is of considerable importance in assessing the potential for harm to the environment and people resulting from possible accidental releases. AHF has a complex thermodynamic behaviour, including self-association in the vapour phase forming so-called oligomers, and also undergoing exothermic reaction with water. Thermodynamic models (e.g. refs. [1] and [2]) for the mixing of anhydrous hydrogen fluoride with moist air indicate that for ambient humidities frequently encountered in European climes, AHF releases which are initially heavier-than-air may become buoyant as the cloud disperses (ref. [3]). The influence of such buoyancy generation may be to mitigate the hazard as compared with non-buoyant releases, due to enhanced dilution and possibly also buoyant lift-off of the cloud from the ground.

The occurrence and, in particular, the quantification of the mitigation of AHF clouds due to buoyancy generation is currently very uncertain. Firstly, the generation of buoyancy is based on the predictions of theoretical thermodynamic models which make certain simplifying assumptions which need to be validated. Secondly, the current understanding of the dispersion behaviour of ground based buoyant releases is somewhat limited, even for buoyancy conserving plumes, let alone for plumes with changing buoyancy. Thirdly, there is a lack of field data for releases of AHF under humid conditions with which to validate models. The EC URAHFREP research programme, of which the work presented in this report is a part, is aimed at reducing these uncertainties to enable more reliable estimates to be made of the hazards associated with accidental AHF releases.

In this report we are concerned with modelling the behaviour of buoyant plumes from continuous ground-based sources. Existing published simple models for the lift-off of initially ground-based buoyant plumes have been reviewed elsewhere (ref. [4]). Plume rise from elevated sources (e.g. from chimney stacks) have been studied extensively for a number of years (e.g. Briggs ref.[5]). The methods used for elevated sources are also expected to be applicable to ground based sources having sufficient buoyancy to rapidly lift clear of, and consequently having little interaction with, the ground. However, what constitutes sufficient buoyancy in this context is yet to be defined, and quantifying this is one of the aims of the current study. Some of the reviewed simple lift-off models are inappropriate for non-buoyancy conserving flows, whilst others, which are potentially applicable, have received only limited, if any, validation for their buoyant lift-off behaviour. Most of the current understanding of lift-off behaviour is based on a few small scale wind-tunnel studies. Although most wind-tunnel studies are implicitly for buoyancy conserving flows, we view the ability of models to predict the dispersion behaviour of these simpler flows as being an important first step to modelling non-buoyancy conserving flows such as those involving AHF. Comparisons of simple buoyant plume lift-off models with existing and new data have been undertaken and are discussed in later sections of this report.

An important aspect not well covered by the existing data is the influence of spatial distribution of buoyancy on lift-off behaviour. This is relevant to modelling AHF plumes, since at the point of becoming positively buoyant they may be wide and shallow due to the initial heavier-than-air behaviour. With this in mind, small-scale wind-tunnel studies of dispersion of continuous plumes from ground-based buoyant sources has been undertaken for

URAHFREP by Hall and Walker (ref. [7]). It is planned that the data from these experiments will be made available as part of the REDIPHEM database. The wind-tunnel studies of Hall and Walker (ref. [7]) systematically vary the source geometry and buoyancy flux and provide on-axis ground level concentrations down-wind and also vertical concentration profiles at two down-wind distances. These data have been used in our model development as reported below.

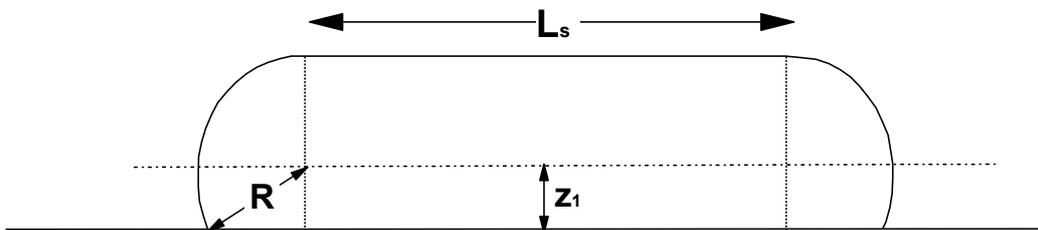
Although the URAHFREP project is concerned mainly with understanding the dispersion and effects of AHF releases, the modelling of dispersion from a ground-based buoyant source is of much wider interest. Other situations where this work may be applicable include the dispersion of pollutants from fires and the dispersion of vapours generated from liquid ammonia and LNG pools.

## 2 A Simple Plume Lift-Off Model

Let us consider a very simple (isothermal) plume model, based on the integral model of Slawson and co-workers (ref. [8]). Our aim is to write down a very simple model which can be used to guide the development of a more “serious” lift-off model.

The plume is modelled as having a “lozenge” shape lateral cross-section consisting of a fixed length segment with semi-circular ends. Although somewhat artificial, this construct has the advantage that it can represent a range of source geometries, including circular (segment length of zero) and line (segment length much longer than semi-circle radius). The imposition of semi-circular ends and fixed length rectangular part means that only the end radii grow as the plume cross-section increases, ultimately leading to a near axi-symmetric plume. Ref. [8] adopted a plume model with such a cross-section to model the dispersion of buoyant material from a row of stacks.

The model of ref. [8] was for a free buoyant plume. We are interested in extending this to a plume which may be in contact with the ground. This is catered for by truncating the cross-section at ground level (see Figure 1).



**Figure 1. Truncated lateral cross-section of the plume (normal to plume trajectory)**

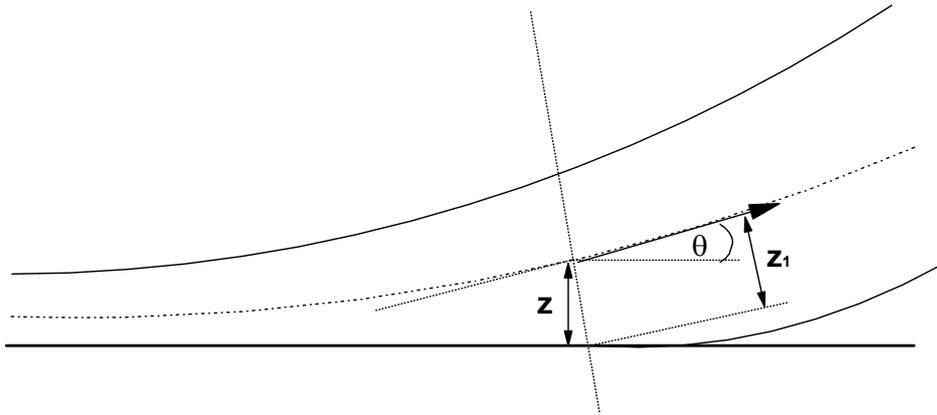
The area of the cross-section is given by

$$A = L_s (R + z_1) + R^2 (\pi - \cos^{-1}(z_1 / R)) + z_1 \sqrt{R^2 - z_1^2}$$

where  $R$  is the radius of the semi-circular end,  $L_s$  is the fixed length of the central section and  $z_1$  is the distance from the ground to the nominal centre of the semi-circle as measured in the plane of the cross-section.  $z_1$  is related to the height of the centre (not strictly the plume centroid)  $z$  by the simple geometric relation

$$z_1 = z / \cos\theta$$

where  $\theta$  is the angle the plume normal makes to the horizontal (Figure 2).



**Figure 2. Orientation of plume cross-section**

The length of the cross-section along the ground is

$$L_{gr} = L_s + 2\sqrt{R^2 - z_1^2}$$

and the length of the free perimeter is

$$L_{free} = L_s + 2R\sqrt{\pi - \cos^{-1}(z_1 / R)}$$

Apart from the above geometric considerations, we shall otherwise model the plume as though it were a free elevated plume. The following equations specify the model:

Contaminant Mass Flux,  $m_g$ :

$$\frac{dm_g}{ds} = 0$$

Total Mass Flux,  $m$ :

$$\frac{dm}{ds} = E_a = L_{free} \rho_a [\alpha |u - u_a \cos \theta| + \beta |u_a \sin \theta| + \gamma u_a]$$

Horizontal Momentum Flux,  $M_x$ :

$$\frac{dM_x}{ds} = E_a u_a$$

Vertical Momentum Flux,  $M_z$ :

$$\frac{dM_z}{ds} = g(\rho_a - \rho)A$$

Plume Centroid Trajectory:

$$\frac{dx}{ds} = \cos\theta \qquad \frac{dz}{ds} = \sin\theta$$

With fluxes defined by

$$m = \rho u A$$

$$m_g = cm$$

$$M_z = mu \sin\theta$$

$$M_x = mu \cos\theta$$

The above equations together with an equation of state to relate density to concentration, and the ambient velocity profile  $u_a$ , may be solved for the evolution of the plume. Based on existing free plume models [5, 6] we select the following entrainment coefficients

$$\alpha = 0.1 \quad \beta = 0.6 \quad \gamma = 0.1$$

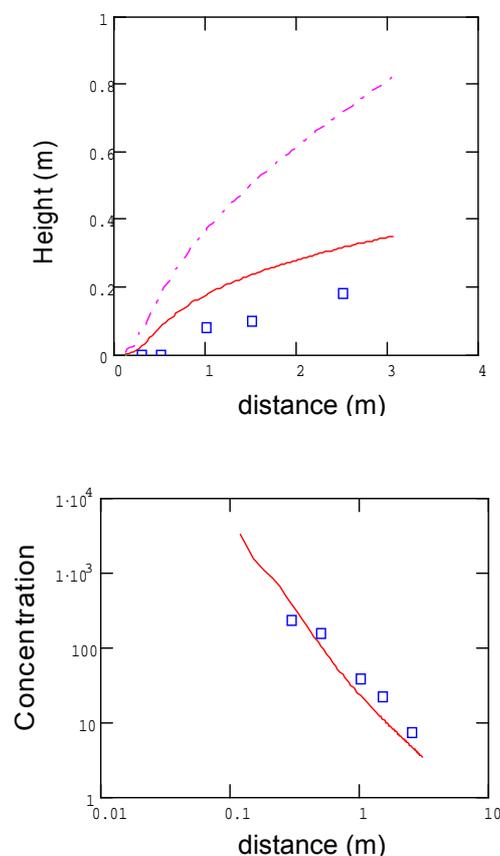
It is acknowledged that the above model is very crude, particularly in its modelling of ambient turbulence by the  $\gamma u_a$ , and (apart from ambient turbulent ‘entrainment’) the neglect of the influence of any velocity shear or influence of the ground. We shall assume, at least in the first instance, that these are relatively unimportant compared with the other influences which are modelled.

The above model has been implemented and solved using MathCad ref.[9] and subsequently also in C++.

### 3 Comparison of the Simple Model with Experimental Data

#### 3.1 DATA OF POREH AND CERMAK (1986)

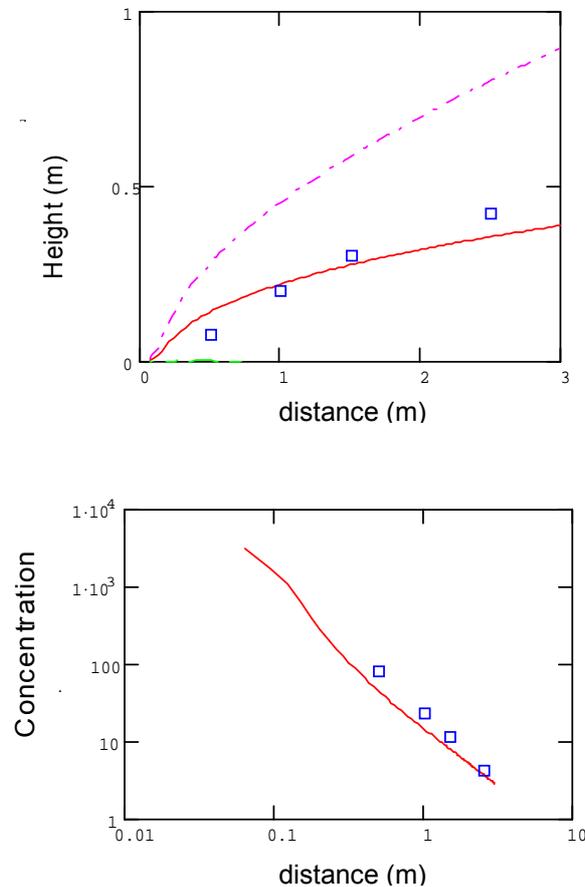
Poreh and Cermak (ref. [10]) conducted experiments on buoyant releases covering a range of conditions, including also releases with significant source momentum. The releases were horizontal in the downwind direction, through a circular orifice with bottom edge at ground level. Ref. [10] classified releases according to non-dimensional buoyancy and momentum fluxes.



**Figure 3. Plume trajectory and maximum concentration as a function of downwind distance from source. Simple model predictions compared with Poreh and Cermak Test 2.1. Squares represent experimental data. The solid line on the trajectory graph is the centreline trajectory. Dashed lines represent the upper and lower “edges” of the plume as determined by the model bulk average  $R$ .**

Comparisons of the above simple model with two tests from ref. [10] are shown in Figure 3 and Figure 4. These tests have been selected as having low momentum fluxes. To enable comparison of model concentrations (plume average) with the centreline concentrations of Poreh and Cermak, we have multiplied the average concentrations by a factor of two. A

factor of two is obtained by equating fluxes in terms of average variables to those from integrated Gaussian profiles in the limit of an ambient density jet in still air (this also assumes equal profile radii for contaminant and velocity).

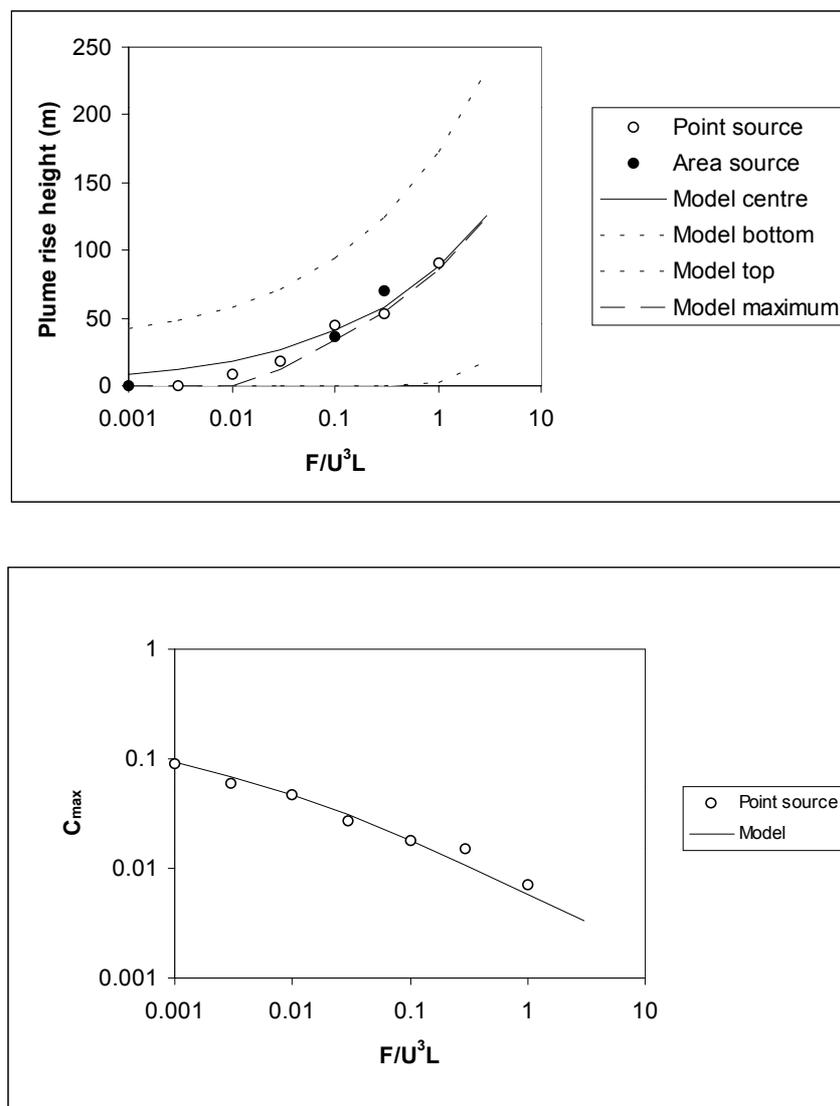


**Figure 4. Plume trajectory and maximum concentration as a function of downwind distance from source. Simple model predictions compared data from Poreh and Cermak Test 5.1. . Squares represent experimental data. The solid line on the trajectory graph is the centreline trajectory. Dashed lines represent the upper and lower “edges” of the plume as determined by the model bulk average  $R$ .**

The simple model predictions are in reasonably good agreement with the Poreh and Cermak data for the tests 2.1 and 5.1 illustrated in Figure 3 and Figure 4. The bottom and top of the plume as defined by the simple model cross-section are shown in the figures. The bottom of the plume is generally at or below ground level (although due to the vertical plume profiles most of the plume is above ground and could be deemed to have lifted off, based on the reduction in ground level concentration).

### 3.2 DATA OF HALL ET AL (1995)

Hall and co-workers conducted a series of experiments investigating buoyant plumes from warehouse fires (ref. [11]). Although most experiments were for releases from model warehouses, some experiments were conducted in the absence of the warehouse for a source flush with the ground. Concentration profiles were measured on a vertical array at a fixed downstream distance. Hall et al. (1995) fitted reflected Gaussian profiles to the data and reported the variation of maximum concentration and its height as a function of a non-dimensional buoyancy flux (the momentum flux for these releases was stated as having negligible influence on the results).



**Figure 5. Plume trajectory and maximum concentration compared with the simple model predictions for the case of no building. Values at a scaled up distance of 300m, with non-dimensional momentum flux of 0.001, as defined in ref. [11].**

Their results indicate that, for these low-momentum releases, the dominant parameter determining plume behaviour is the non-dimensional buoyancy flux, defined as

$$\frac{F}{u^3 L}$$

where

$$F = \frac{1}{\pi} g \frac{(\rho_a - \rho_s)}{\rho_a} \dot{V}_s$$

is the conserved buoyancy flux (note this definition includes a  $1/\pi$  factor which simplifies axisymmetric equations) and  $u$  and  $L$  are fixed reference velocity and length scales.

Comparison of our simple model with the data of Hall et al is shown in Figure 5. In addition to the model “centre” height prediction is the model “maximum” concentration height prediction. This results from assuming the model trajectory equations relate to the centroid of an assumed Gaussian distribution in the vertical and from equating the model average area with an integral over Gaussian profiles in the limit of an ambient density plume moving at the ambient windspeed:

$$m_g = c \rho_a u_a A = c_{\max} \int_{-\infty}^{\infty} dy \int_0^{\infty} dz \exp\left[-\frac{(z - z_{\max})^2}{2\sigma_z^2}\right] \exp\left[-\frac{y^2}{2\sigma_y^2}\right]$$

then assuming

$$c = c_{\max} / 2$$

gives

$$A = 2\pi\sigma_y\sigma_z \left[1 + \operatorname{erf}(z_{\max} / \sqrt{2}\sigma_z)\right] \quad (1)$$

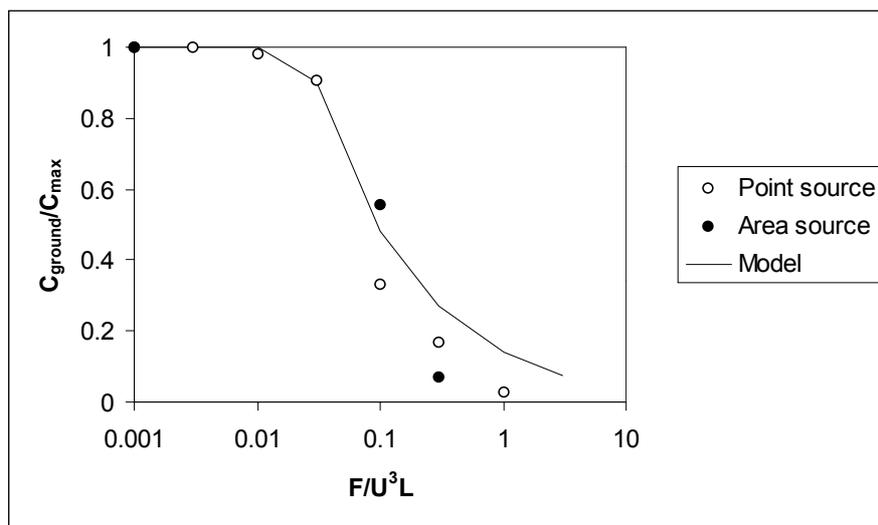
To extract  $\sigma_z$  from the above we appeal to the assumed axis-symmetry in the model (but not necessarily the data) and set  $\sigma_y = \sigma_z$ . This assumption is very much open to question. The maximum height  $z_{\max}$  is related to the centroid height  $\bar{z}$  by

$$\bar{z} = \frac{\int_0^{\infty} dz z \exp\left[-\frac{(z - z_{\max})^2}{2\sigma_z^2}\right]}{\int_0^{\infty} dz \exp\left[-\frac{(z - z_{\max})^2}{2\sigma_z^2}\right]}$$

giving

$$\bar{z} = z_{\max} + \sqrt{\frac{2}{\pi}} \sigma_z \frac{\exp\left[-\frac{z_{\max}^2}{2\sigma_z^2}\right]}{\left[1 + \operatorname{erf}\left(z_{\max} / \sqrt{2}\sigma_z\right)\right]}$$

which must be solved together with equation (1) to give  $z_{\max}$  and  $\sigma_z$ . If the resultant  $z_{\max}$  is predicted to be lower than the ground it is set to zero (corresponding to ground-level). This rather ad hoc method of introducing profiles has the desired (and physically reasonable) effect of predicting a lower  $z_{\max}$  when the plume is on the ground. When the plume has lifted off  $z_{\max}$  tends towards  $\bar{z}$ . In addition with  $\sigma_z$  estimated we may compare the predicted ratio of ground level concentration divided by maximum concentration with the observed values. This comparison is illustrated in Figure 6 below. The agreement, albeit with some rather ad hoc assumptions about plume cross-section and profiles, is remarkably good. The agreement for high buoyancies is to be expected since the data and model both show the expected asymptotic plume rise with buoyancy. However, the agreement at lower buoyancies is encouraging.



**Figure 6. Ground level concentration divided by maximum concentration at a fixed downwind distance as a function of non-dimensional buoyancy flux.**

### 3.3 EXPERIMENTS OF HALL AND WALKER (2000)

As already mentioned, new wind-tunnel experiments as reported by Hall and Walker (ref. [7]) have been conducted as part of the URAHFREP project. Hall and Walker adopted a horizontal ground-level area source (similar to that used in ref.[11]) designed to minimise the influence of source momentum on the plume.

The results of these experiments were again found to be dependent on the non-dimensional buoyancy flux as defined above. In these experiments Hall and Walker use  $L=6.7\text{cm}$  with  $u$

as the wind speed at the height  $L$ ; these reference scales are somewhat arbitrary (from the mathematical modelling point of view), but are useful in scaling the wind-tunnel results. The roughness length,  $z_0$  was estimated to be approximately  $z_0 = 0.03L$ . Hence, scaling the reference length to 10m (a 150 times scaling) corresponds to a site with  $z_0 = 0.3m$ .

Hall and Walker found that increasing the width of the source (with the same total buoyancy flux) decreases plume rise, whereas plume rise was enhanced for long narrow sources aligned along the wind. In accordance with earlier studies Hall and Walker found concentration maximum remained at ground level until a critical buoyancy was reached, after which increasing the buoyancy increased the height of the concentration maximum and rapidly decreased the ground level concentration. This lift-off behaviour is reasonably well correlated by the dimensionless buoyancy per unit source width  $W$

$$\frac{F}{u^3W}$$

Hall and Walker quote that, for distances  $15L$  and  $30L$  downwind from the leeward side of the source, the first onset of plume rise, where the concentration maximum rises from ground, occurs when  $\frac{F}{u^3W} \sim 0.01$ , and lift-off, where the ground level concentration has fallen to 10-

20% of the maximum occurs when  $\frac{F}{u^3W} \sim 0.035$ . These values are only approximate and

there is considerable scatter in their correlation. In particular, enhanced plume rise was observed for the long sources aligned along the wind direction, with some shreds of plume material being observed as remaining near ground level.

Comparisons of the data of Hall and Walker with our simple lift-off model are shown in Figure 7 and Figure 8 below. The source sizes used in the experiments are given in Table 1.

Firstly we consider the decay of maximum concentration as predicted by the model. As for the other experiments this is relatively well predicted by the model, although there is a tendency for the model to over-predict concentrations for the larger area sources at low buoyancy fluxes. The dependence on buoyancy flux and on source size is relatively well represented. We have not attempted to use the model for long sources aligned in the direction of the wind. This is because the assumption of the model for bending of the source is particularly dubious there. We discuss an alternative approach later in this report.

Consider next the predicted position of the concentration maximum. For the small square sources, which most readily lift-off, the model is in good agreement with the data. The model shows little dependence on source size, whereas the data shows a more marked effect. The differences are even more exaggerated for the wide sources which do not lift-off so readily. The simple model does not fully capture this apparent suppression of plume lift-off for wide sources.

**Table 1. Source Sizes Used in the Experiments of Hall and Walker (2000)**

<b>Group</b>	<b>Identifying Letter</b>	<b>Width y/L</b>	<b>Length x/L</b>	<b>Area xy/L<sup>2</sup></b>
<b>“Square” Sources</b>	A	0.448	0.448	0.2
	B	1.19	1.19	1.4
	C	2.69	2.69	7.2
	D	3.43	3.43	11.8
	E	6.87	7.16	49.2
<b>“Wide” Sources</b>	G	0.448	3.43	1.54
	H	1.19	3.43	4.10
	D	3.43	3.43	11.78
	I	7.16	3.43	24.59
	J	14.33	3.43	49.19
	K	28.66	3.43	98.37
<b>“Long” Sources</b>	F	3.43	0.448	1.54
	L	3.43	1.19	4.10
	D	3.43	3.43	11.78
	M	3.43	7.16	24.59
	N	3.43	14.33	49.19
	P	3.43	28.66	98.37

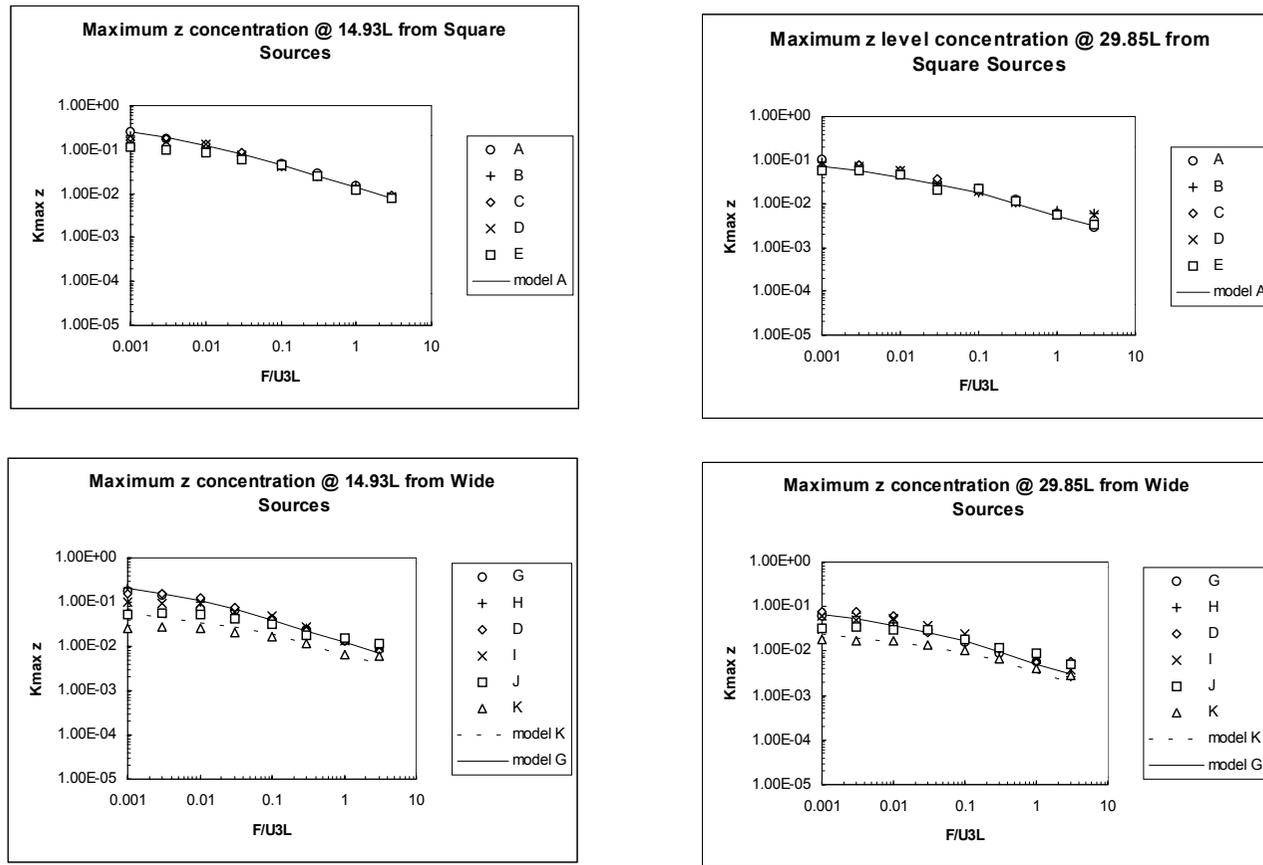


Figure 7. Comparison of simple model predictions for maximum concentration with the data of Hall and Walker (2000) for square and wide sources.

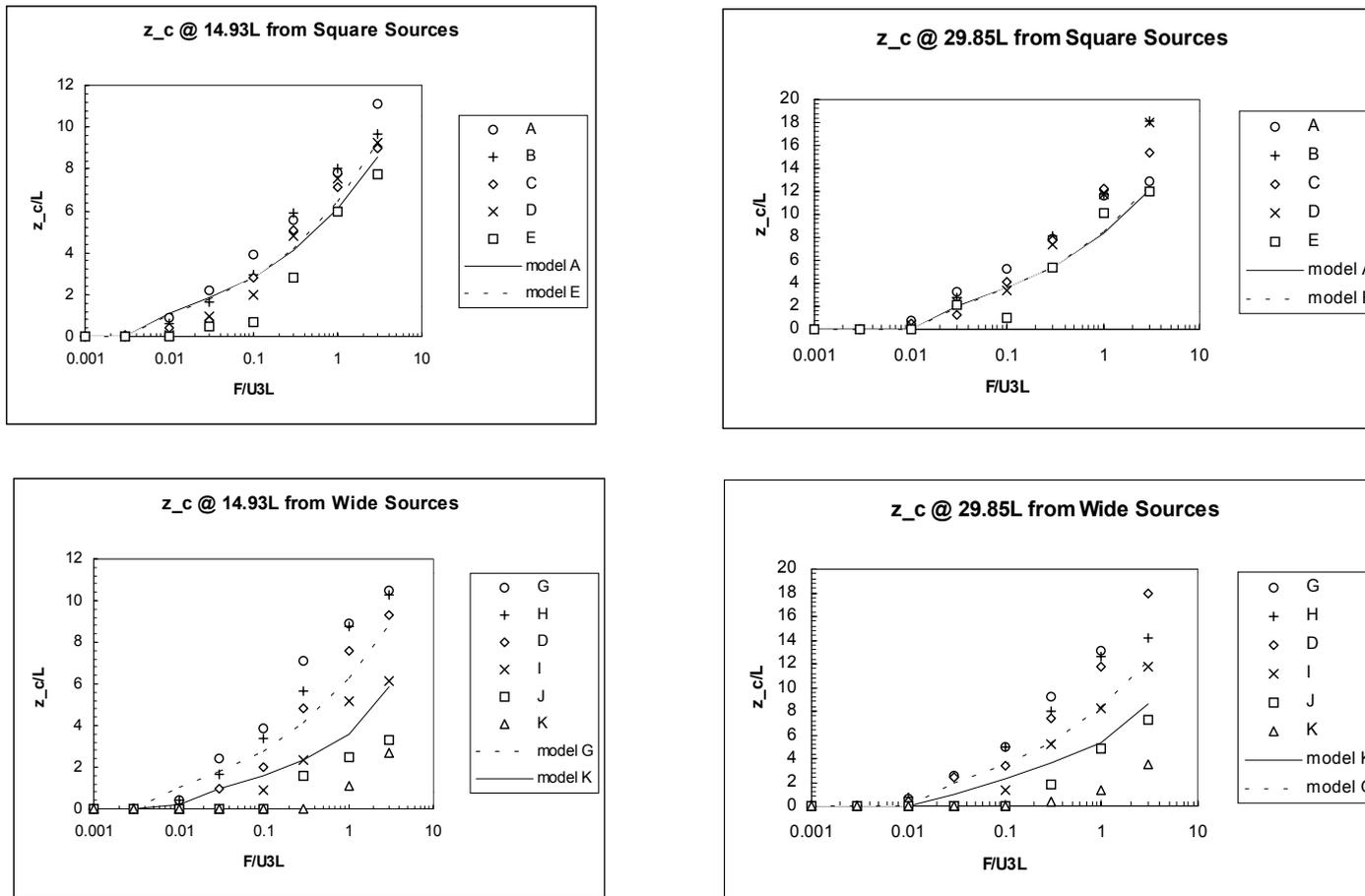


Figure 8. Comparison of simple model predictions for trajectory of maximum concentration with the data of Hall and Walker (2000) for square and wide sources.

## 4 Further Modelling Investigations

We have described comparisons of a simple integral model with experimental data on ground-based buoyant releases. It would appear from these comparisons that simple models of the type described are capable of giving a good representation of the dilution behaviour of such releases. The lift-off behaviour is reasonably well represented for “small” sources, whereas for “wide” sources they under-predict plume lift-off. This begs the question: “small” or “wide” compared with what? For a buoyant release in a wind, on dimensional grounds, one expects the relevant length scale to be the buoyancy length scale formed from the conserved buoyancy flux and the wind speed. This is confirmed by the work of Hall and Walker (2000) in their suggested non-dimensional critical lift-off parameters.

If it were possible to use such critical lift-off parameters directly in the modelling of non-buoyancy conserving flows, then this could have a number of attractions. Using such criteria might improve on the estimation of when lift-off starts to occur, especially for wide, initially ground-based plumes. It also allows the possibility of using a wholly ground based plume model up to the lift-off point, and subsequently an elevated model, possibly similar to the one discussed above, when the lift-off condition is exceeded. Existing, ground-based dense gas dispersion models for AHF such as DRIFT (ref. [12]) could be modified along these lines. However, the grounded plume model would still be required to account for the enhanced vertical mixing prior to lift-off, and the elevated plume model would still require some suppression of plume rise to be in agreement with the wind-tunnel data for lift-off of wide sources. Consideration is given to these factors in the following sections.

### 4.1 CRITICAL LIFT-OFF PARAMETER

The data of Hall and Walker (ref. [7]) indicate the range of non-dimensional buoyancy fluxes over which the ground-level concentrations decrease due to plume rise. As we have already seen we can equivalently regard this as a dependence on the Richardson number,  $Ri_*$ . Using the same method of estimating  $Ri_*$  as used previously for determining enhanced dilution, and with the same caveats, we deduce the following based on the report by Hall and Walker [7]:

- The first onset of plume rise, defined by the point where the concentration maximum leaves the ground, occurs for  $F/u^3W \sim 0.01$ , which is equivalent to the point where  $Ri_* \sim 2$ .
- The ground level concentration falls to 10-20% of the maximum when  $F/u^3W \sim 0.035$ , equivalent to  $Ri_* \sim 10$
- For  $F/u^3W > 0.3$ , equivalent to  $Ri_* > 70$ , the ground level concentration is generally less than 5% of the maximum.

The above values are consistent with Briggs' earlier estimates of critical lift-off parameter [17, 18].

## 4.2 ENHANCED MIXING OF BUOYANT GROUND-BASED PLUMES

We have seen that the simple plume model reproduces quite well the maximum concentration behaviour, even while the maximum is on the ground. This can be used to guide our choice of modification to entrainment to account for enhanced vertical mixing of ground-based buoyant plumes. In the simple model, the main influence of buoyancy on dilution is through the second entrainment term. In the bent-over plume limit, where the plume is moving horizontally at approximately the wind speed, the buoyant entrainment term is proportional to the vertical component plume velocity,  $u_z$ . In terms of the entrainment velocities we can write:

$$u_{\text{entrain}} = u_{\text{entrain}}(\text{passive}) \left[ 1 + \lambda \frac{u_z}{u_a} \right]$$

where  $u_{\text{entrain}}(\text{passive})$  is the passive entrainment velocity corresponding to ambient turbulence. If we consider the case of a two-dimensional buoyant plume on the ground, then on dimensional grounds we can show

$$\frac{u_z}{u_a} \propto \left[ \frac{F_W}{u_a^3} \right]^{1/2} \propto Ri_*^{1/2}$$

where  $F_W$  is the buoyancy flux per unit width and  $Ri_*$  is a Richardson number for the plume based on the layer depth,  $H$ , density deficit,  $\rho - \rho_a$  and atmospheric friction velocity  $u_*$ :

$$Ri_* = g \frac{(\rho_a - \rho)H}{\rho_a u_*^2}$$

We shall attempt to determine an effective entrainment velocity from the ground level concentration measurements of Hall and Walker (ref.[7]) for their widest source, which most closely approximates to a 2-dimensional source and also remains grounded over the widest range of buoyancies.

Following Britter (ref.[13]) we may define an effective depth,  $H$ , of the two-dimensional plume from the ground-level concentration,  $c_{\text{max}}$ , and an assumed reference velocity,  $u$ , for the plume:

$$c_{\text{max}} u H = Q_W$$

where  $Q_W$  is the source rate per unit width (we shall assume volumetric concentrations here). In terms of the non-dimensional concentration,  $K_{\text{max}}$ , defined by

$$K_{\max} = \frac{c_{\max} u L^2}{Q_w W}$$

of Hall and Walker (2000), assuming the same reference scales are chosen, we find that

$$\frac{H}{L} = \frac{1}{K_{\max}} \frac{L}{W}$$

As discussed in ref. [13] we can define an effective entrainment velocity,  $u_e$  from the derivative of  $H$  with respect to downwind distance  $x$ :

$$\frac{u_e}{u} = \frac{dH}{dx}$$

We estimate  $dH/dx$  for each non-dimensional buoyancy flux from the gradient of a linear fit to the data of  $H/L$  versus  $x/L$  as derived from Hall and Walker's ground level centreline concentration measurements (for buoyancies where the maximum is still at ground level at the two vertical arrays). A better linear fit of  $H/L$  with distance was obtained by excluding the nearest two measurements to the source. For the neutral buoyancy condition it was necessary to exclude the nearest three measurements to the source. These data points are excluded from the following analysis.

Rather than consider the behaviour of  $u_e/u$  as a function non-dimensional buoyancy flux ( $F/u^3 W$ ), we prefer to consider  $u_e/ku_*$  as a function of  $Ri_*$  which is estimated from

$$Ri_* = \left( \frac{u}{u_*} \right)^2 \frac{\pi F}{u^3 W}$$

and

$$u_* = k u / \ln(L/z_0)$$

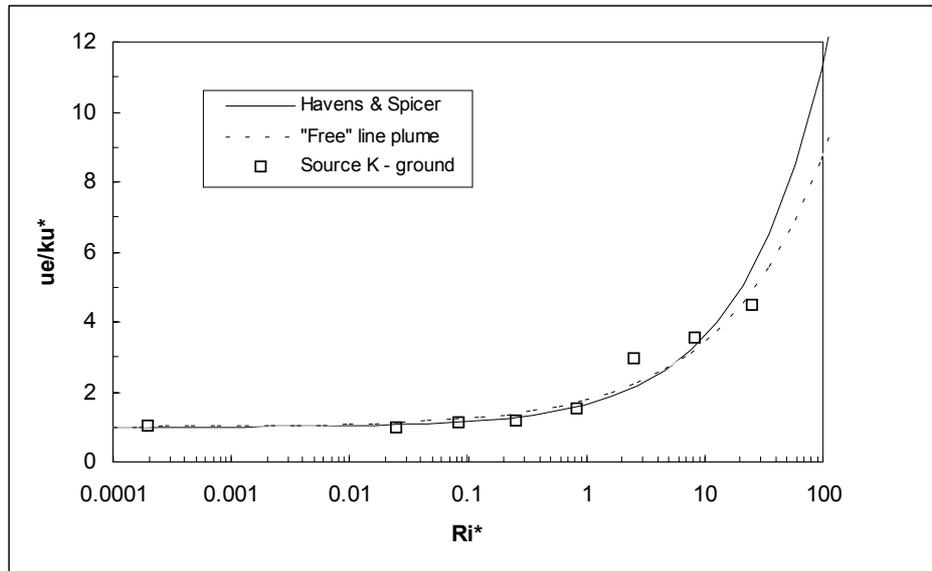
where  $k$  is the von Karman constant having a value of approximately 0.4. The quantity  $u_e/ku_*$  is of interest because it is approximately one for passive entrainment. Figure 9 shows the resultant data points from an analysis of the data of Hall and Walker (2000). Also included on Figure 9 are the correlation used by Havens and Spicer (1990) ref. [14] as originally suggested by Colenbrander and co-workers (ref. 15) for buoyant clouds:

$$\frac{u_e}{ku_*} = \left( 1 + 0.65 Ri_*^{0.6} \right)$$

and, based on the analytic solution of our simple model for a two dimensional line source, adopting an entrainment coefficient  $\beta = 0.6$  (ref. [5])

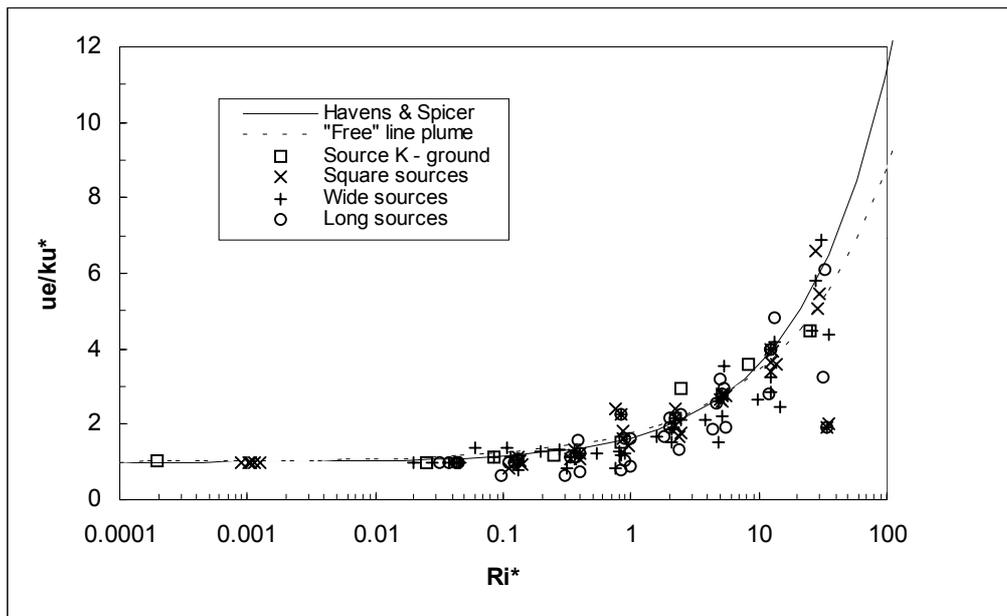
$$\frac{u_e}{ku_*} = (1 + 0.77 Ri_*^{1/2})$$

This latter correlation is denoted “Free” line plume on Figure 9. It is seen from Figure 9 that both these approaches are in reasonably good accord with the wind-tunnel data and are numerically similar over the range of Richardson numbers covered by the data.



**Figure 9. Entrainment as a function of Richardson number for positively buoyant ground-based widest “line” source in the experiments of Hall and Walker (ref.[7]).**

It is more problematical to repeat this analysis for the other sources, since they tend to lift-off at lower buoyancies, and when lifted off, the ground level sensors no longer record the maximum concentration. Because of the earlier plume lift-off, and the non two-dimensional nature of the plumes from these other sources, they are suited to isolating a top-entrainment velocity. The result of incorporating these additional data are shown in Figure 10. For these cases the maximum concentration must be obtained from the two vertical arrays, so there are only two downstream distances with which to determine the  $dH/dx$  derivative. In Figure 10 we show the combined data from all sources. For this plot the data have been normalised to be one at the lowest buoyancy condition. This is taken as an indication that the passive term  $u_e/ku_*$  needs to be modified to fit some of the area sources. Not surprisingly Figure 10 shows considerably more scatter than Figure 9.



**Figure 10. Entrainment as a function of Richardson number for positively buoyant ground-based sources – all sources from the experiments of Hall and Walker (ref.[7]).**

In the above we have conveniently glossed over the fact that the definition of entrainment velocity is dependent upon the chosen reference velocity. In our opinion the analysis is most relevant if the chosen velocity scale corresponds to the mean advection velocity of the plume. The choice of reference length and hence velocity in the experiments of Hall and Walker means that, over the distances and buoyancies covered, the reference velocity scale might not be too different from the mean advection velocity.

### 4.3 IMPROVED MODELLING OF AREA SOURCES OF HALL AND WALKER (2000)

As already mentioned, the experiments of Hall and Walker (ref.[7]) involve releases through horizontal area sources flush with the wind-tunnel floor. Some of the source sizes are comparable to the downwind distances over which concentration measurements were obtained. Clearly for these large sources some account of dilution and trajectory over the source is desirable. The simple integral model given in the preceding sections is not ideally suited to this. The simple model is not valid when the plume bends strongly in the wind, as is the case for many of the releases in the studies of Hall and Walker (2000). The model may predict overlapping plume cross-sections and takes no account of the curvature of the plume implying that the upwind edge sweeps out a longer distance than the downwind edge. The good agreement between model and earlier experiments is probably attributable to this source region being of little significance compared with the subsequent dispersion. The experiments of Hall and Walker (2000) are more sensitive to the source modelling because the dimension of many of the sources is significant compared with the downstream distances. It is appropriate to attempt to improve on the modelling of the dispersion from such area sources, since the relatively poor performance of the simple model for large sources may be partly due to inadequate representation of the area source.

An alternative approach to modelling the source region is to treat the released material as being taken up from the source and transported through a vertical source window. In the passive limit, standard passive area source approaches can be applied (e.g. ref. [16]).

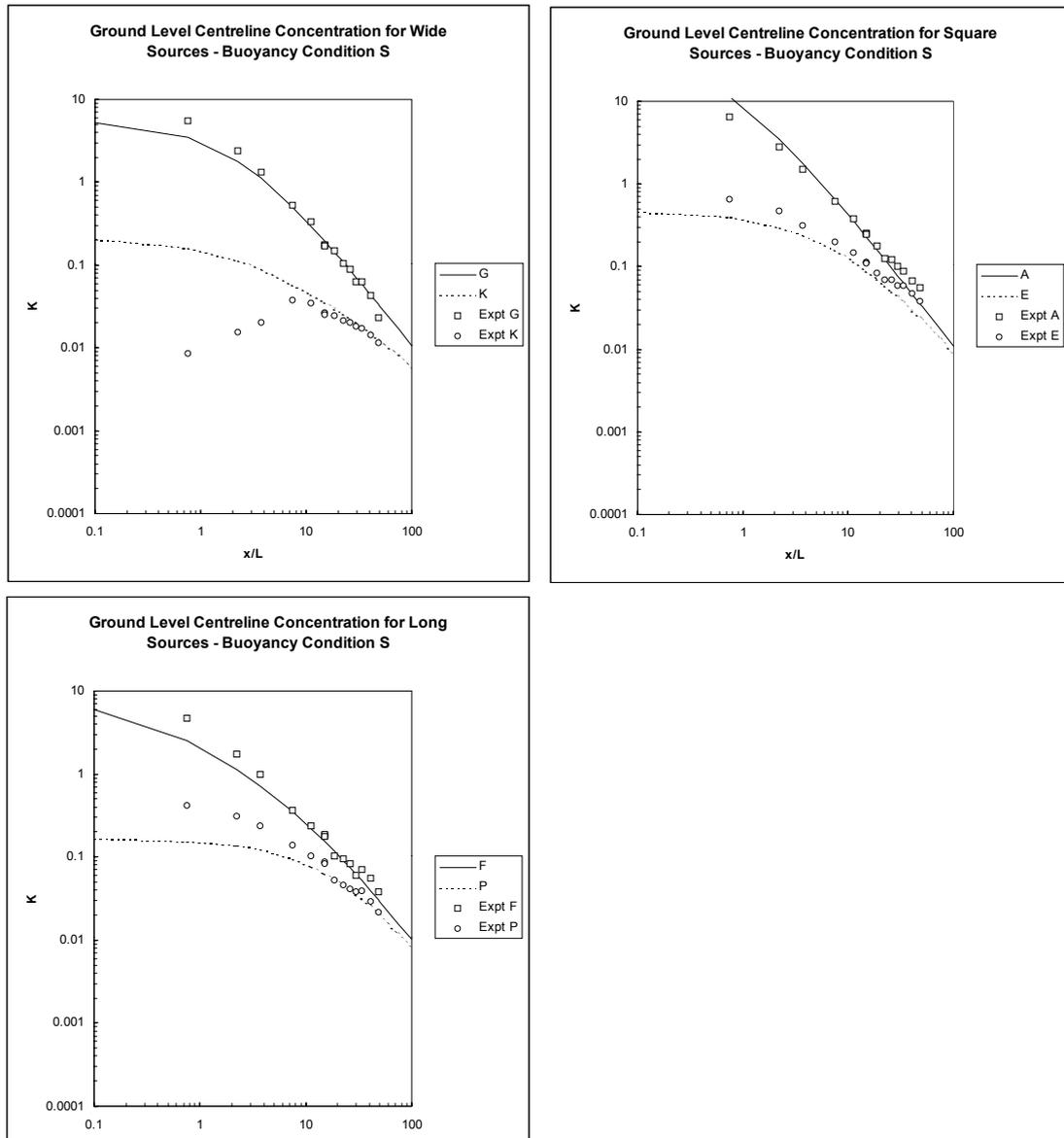
We have adopted the following approach to modelling the sources of Hall and Walker:

1. The square sources have been modelled as point sources originating in the centre of the source.
2. The wide sources have been modelled as line sources.
3. The long sources have been modelled as point sources originating from the mid-point of the source.

#### **4.3.1 Passive Sources**

For the effectively passive releases (Buoyancy Condition S), applying the passive model of ref.[16] results in the predictions for ground level concentration shown in Figure 11. These are a marked improvement over the simple plume model predictions for such sources. For the widest source (K) the experimental data indicate that the ground level concentration initially rises downwind from the source before decreasing. This is not observed for the other sources and may be due, in part, to the effectively two-dimensional nature of the source affecting bending of the plume. The correct asymptotic decay appears to be predicted by the passive model.

We are not aware of models for buoyant releases from area sources (except for the highly buoyant situation where the plume rises direct from the source). In these cases we choose to make the same assumptions regarding source approximation as for passive sources. To compare with the wind-tunnel results over a range of buoyancies we shall then consider combining the predictions of passive and buoyant plume models. Such comparisons with analytic solutions of plume models may then give some further insight into how the simple model might be further improved, particularly with regards to improving plume lift-off behaviour for wide sources.



**Figure 11. Ground level centreline concentrations based on passive line and point sources. Buoyancy condition S denotes the passive sources.**

### 4.3.2 Buoyant Sources

For buoyant sources we seek simple model equations for plumes from point and line sources. We consider a plume of uniform rectangular lateral cross-section, of vertical depth,  $H$ , and lateral width,  $W$ . We shall make the Boussinesq approximation, i.e. that the density can be approximated by the ambient density everywhere, except in the buoyancy terms. Also we shall make the “bent-over” plume approximation that the horizontal velocity component can be replaced by the wind speed which is treated as taking a constant value and that this horizontal component is much larger than the vertical velocity component, implying that the plume trajectory is nearly horizontal. When the buoyant entrainment dominates over

entrainment due to ambient turbulence, the mass balance equation becomes for the elevated region

$$\frac{d(HW)}{dx} = 2\beta(W + H)\frac{u_z}{u_a}$$

where  $x$  is the downwind distance,  $u_z$  the vertical component of velocity and  $u_a$  is the wind speed which is treated as constant.

If we further assume that the spreading rates laterally and vertically are equal  $dH/dx \approx dW/dx$  then the mass balance becomes

$$\frac{dH}{dx} = \frac{dW}{dx} = 2\beta \frac{u_z}{u_a}$$

The trajectory is given by

$$\frac{dz}{dx} = \frac{u_z}{u_a}$$

which upon substitution into the mass balance and integrating implies that

$$H - H_0 = 2\beta(z - z_0)$$

where subscript 0 indicates initial conditions.

The vertical momentum equation is

$$\frac{d(HWu_z)}{dx} = g \left( \frac{\rho_a - \rho}{\rho_a} \right) \frac{HW}{u_a}$$

which also can be integrated giving

$$HWu_z = HWu_z|_0 + \pi \frac{F}{u_a^2} (x - x_0)$$

where

$$F = \frac{g}{\pi} \left( \frac{\rho_a - \rho}{\rho_a} \right) u_a HW$$

is the buoyancy flux which is conserved for an isothermal mixture of ideal gases.

Substituting for  $u_z$  in the mass balance equation gives

$$\frac{dH}{dx} = \frac{2\beta}{HWu_a} \left[ HWu_z|_0 + \pi \frac{F}{u_a^2} (x - x_0) \right]$$

Consider the following cases

(i) When  $H \approx W$

$$H = \left( \frac{6\beta}{u_a} \right)^{1/3} \left[ HWu_z|_0 (x - x_0) + \frac{\pi}{2} \frac{F}{u_a^2} (x - x_0)^2 \right]^{1/3}$$

Assuming the plume depth at the source can be neglected and a source position of  $x_0 = 0$ , i.e. a point source at the origin, we have

$$\frac{H}{L} = (3\pi\beta)^{1/3} \left( \frac{F}{u_a^3 L} \right)^{1/3} \left( \frac{x}{L} \right)^{2/3}$$

where we have introduced an (arbitrary) length scale  $L$  to facilitate comparison with the wind tunnel data. The vertical velocity obtained from the vertical momentum equation may then be integrated to give the plume rise (centroid) height as a function of distance:

$$\frac{z}{L} = \frac{3}{2} \left( \frac{\pi}{9\beta^2} \right)^{1/3} \left( \frac{F}{u_a^3 L} \right)^{1/3} \left( \frac{x}{L} \right)^{2/3}$$

Thus the famous 2/3 power law asymptotic plume rise formula is obtained (as must be the case on dimensional grounds). Briggs plume rise entrainment constant of 0.6 corresponds to  $\beta = 0.53$  in the above equations.

The dimensionless average concentration is given by

$$K = \left( \frac{L}{H} \right)^2$$

For centreline concentration the above should be multiplied by a factor of two.

(ii) When  $W \gg H$

Neglecting the source height, this approximation corresponds to a wide line source. By the same approach as for point sources we find

$$\frac{H}{L} = (2\beta)^{1/2} \left( \frac{F}{u_a^3 W} \right)^{1/2} \frac{x}{L}$$

$$\frac{z}{L} = \left( \frac{\pi}{2\beta} \right)^{1/2} \left( \frac{F}{u_a^3 W} \right)^{1/2} \frac{x}{L}$$

$$K = \left( \frac{1}{2\beta} \right)^{1/2} \left( \frac{F}{u_a^3 W} \right)^{-1/2} \left( \frac{W}{L} \right)^{-1} \left( \frac{x}{L} \right)^{-1}$$

Note that the above equations do not take account of the possibly fundamentally different flows for point and line sources.

We wish to compare the above predictions with the wind-tunnel data of ref. [7].

To enable the model to asymptotically approach the passive model dilution at low buoyancies and the buoyant plume dilution at high buoyancies we add the dilutions according to

$$\frac{1}{K} = \frac{1}{K_{passive}} + \frac{1}{K_{buoyant}}$$

A similar approach to adding contributions was adopted by Hanna et al [18]. The results for maximum concentration predicted by the above equations using  $\beta = 0.53$  compared with data of ref. [7] are shown in Figure 12. The inclusion of the area passive source has improved the agreement with data for the low buoyancies. The buoyancy dependence of maximum concentration is quite well represented, apart from a slight tendency for the model to under-predict the dilution for the widest sources at highest buoyancies. The long sources are not represented so well by the equations.

Comparison of plume rise predictions with the data of ref. [7] is again complicated by the model being more appropriate for plume centroid height. Also, for plume rise, it is less straightforward to include the effects of ambient turbulence, than it was for concentration. If we ignore these effects, the plume rise predicted by the above equations is likely to be overestimated, except when the plume has risen sufficiently clear of the ground for the centroid and plume concentration maximum to be in close agreement. Comparisons are shown in Figure 13 as a function of buoyancy flux non-dimensionalised by source width  $W$ . The plume rise from the wider sources is overestimated by the model, whereas the rise from narrower sources are in better agreement.

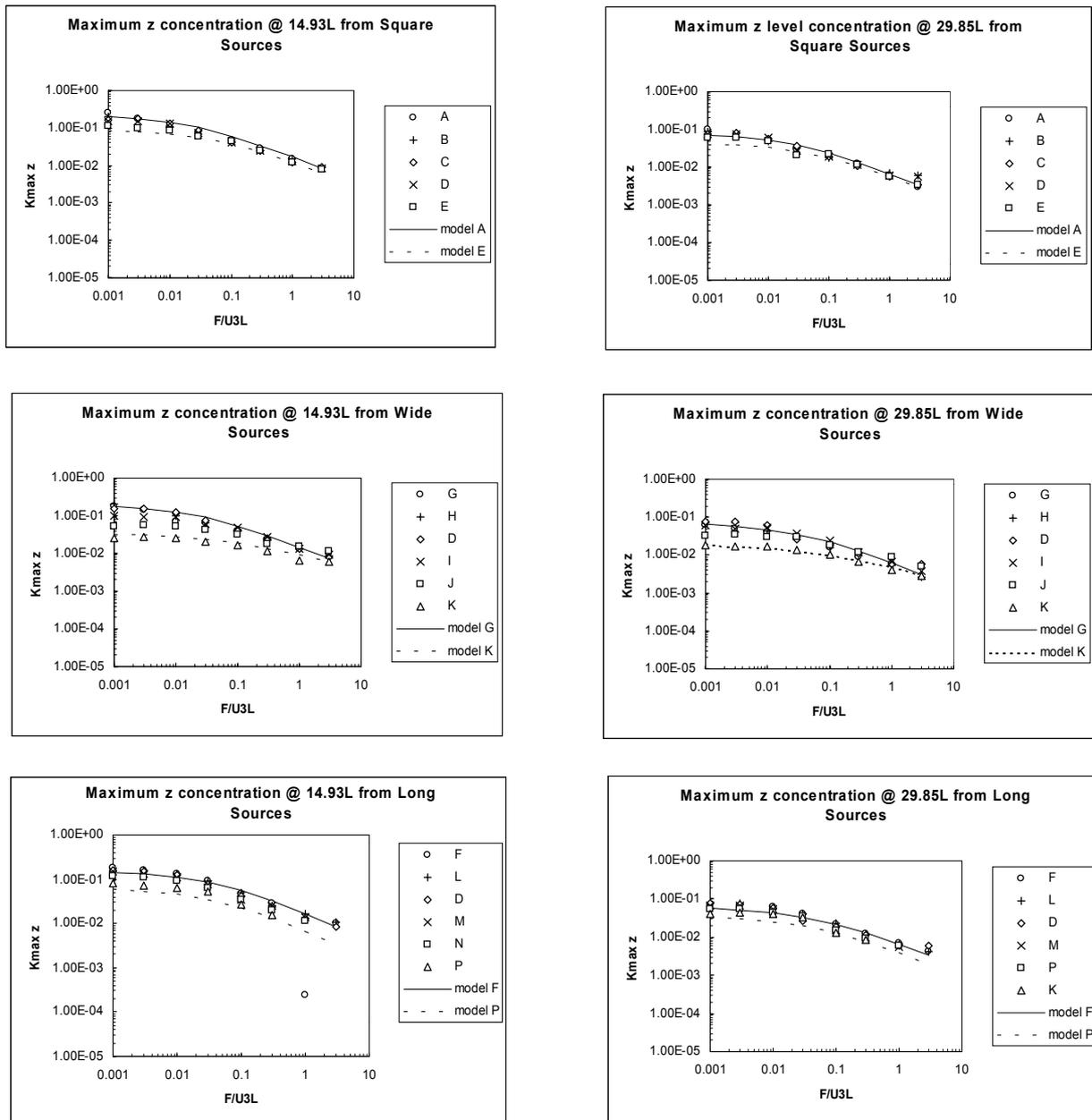


Figure 12. Maximum concentration predictions from point and line sources compared with the data of Hall and Walker [7]

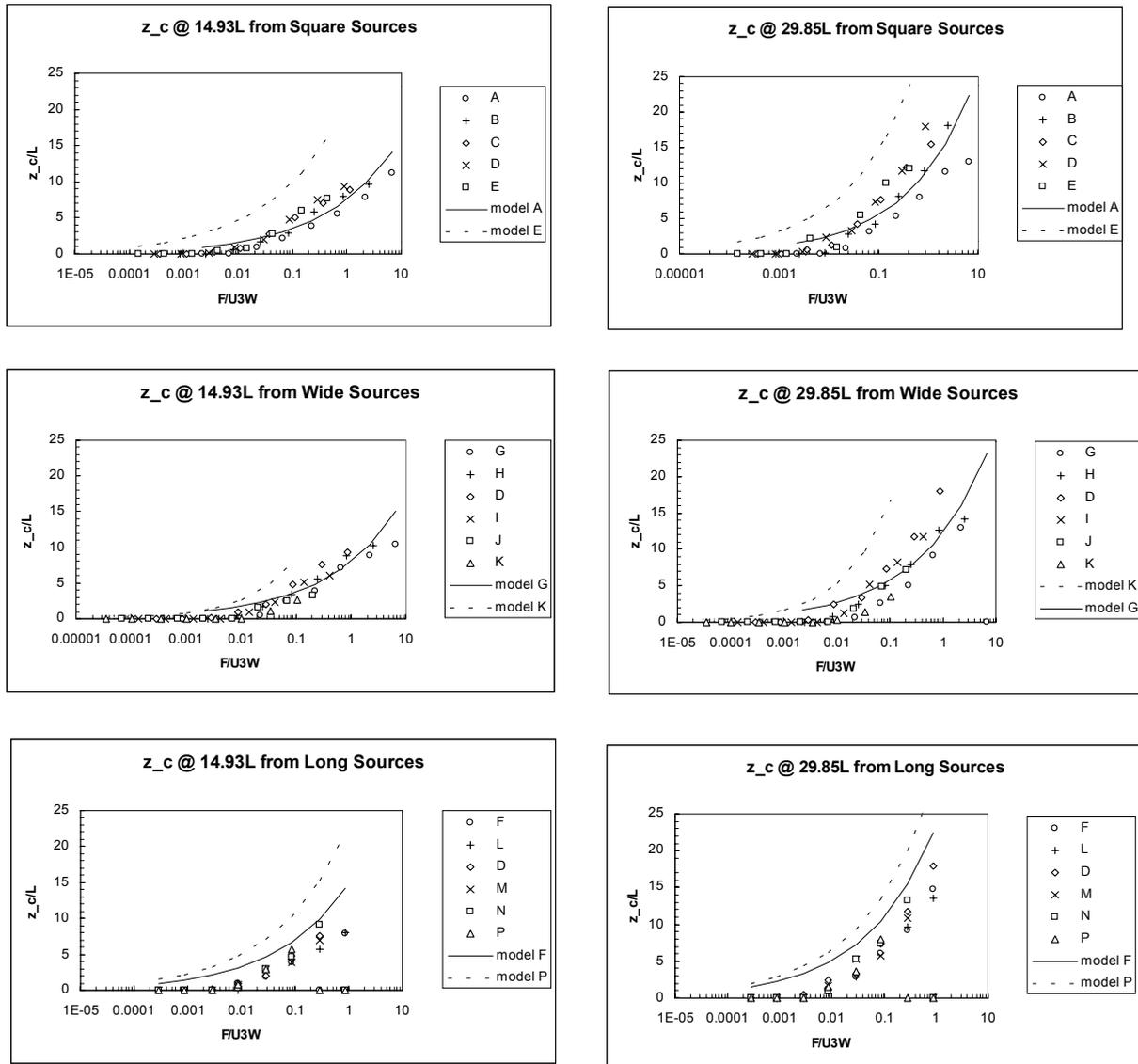


Figure 13. Centroid height predictions compared with measured heights of maximum concentration from ref. [7]

## 4.4 BUOYANT PUFF RELEASES

The modelling efforts in this report relate to steady continuous buoyant plumes. Under the URAHFREP project Hall et al [19] studied short duration buoyant releases. These puff releases were designed to have similar release conditions to the continuous plume, except for their finite duration. Hall et al [19] make a number of interesting observations:

- releases that lead to plumes being grounded also appear to lead to grounded puffs based on mean concentration, but peak concentrations are observed to be above ground
- in the near field, the puff rise appears to be very similar to the equivalent plume rise
- in the far field, puff rise is less than plume rise, presumably due to the influence of longitudinal dispersion
- concentration was observed to scale with volume (like a puff) rather than volume flux and windspeed (for a plume)
- concentrations in the short duration releases are lower than in the equivalent plume releases
- high degrees of intermittency and repeat variability are observed and these variabilities were more pronounced for higher buoyancies

The reader is referred to ref. [19] for details. The detailed implications of these findings for mathematical models have yet to be assessed. It would be interesting to compare the new wind-tunnel results with mathematical models for thermals and plumes. In such a study the scaling of the buoyant rise and dilution predicted by these limiting case models might be compared with the experimental data. The observed high levels of intermittency and high repeat variability may hinder such comparisons. In the mean time, the wind-tunnel study lends support to the statement that modelling releases as though they are steady continuous is pessimistic in terms of concentration estimates.

## 5 Conclusions

A simple plume lift-off model has been presented and compared with experimental data on buoyant plumes from ground level sources. The data cover a range of buoyancies, from near neutral to strongly buoyant where the plume behaves like a classical free plume. The main intention of these comparison studies was to test the “null” hypothesis that the ground has very little influence on plume rise. This appears to be a reasonable approximation for “small” sources which appear to most readily lift-off and give closest agreement with the simple model. However, the recent experiments of Hall and Walker indicate significant suppression of plume rise for wide sources, above that predicted by the simple model. This suppression is grossly underestimated by the model for the widest (K) source. This result suggests that models making similar plume rise assumptions to our simple model are likely to overestimate plume rise and hence underestimate ground level concentrations for wide sources. Surprisingly, the entrainment model for the simple model appears to predict remarkably well the maximum concentration for most of the sources, even though the plume rise might be in error.

We have found it difficult to improve on the estimation of plume rise from wide sources, for example by introducing empirical corrections to the vertical momentum equation, without detrimentally affecting the dilution and rise predictions for the other sources. This is because to suppress plume rise, the vertical component of plume velocity must be reduced. However, reducing the vertical velocity component also has the undesirable effect of reducing dilution of the buoyant plume as controlled by the adopted entrainment formula. As well as spoiling the good agreement with data for dilution, suppression of entrainment acts against suppression of plume rise by virtue of the more concentrated plume and hence larger buoyancy force at a given distance. Modifying the entrainment formula to depend upon a buoyant velocity scale rather than vertical component of velocity may circumvent such problems and may be worth investigation in future studies.

Due to the difficulties of the simple model in predicting plume lift-off from wide sources, some consideration has been given to the use of a grounded plume model up to some defined lift-off buoyancy, and then subsequently using a plume rise model. These issues are discussed below.

The sources used by Hall and Walker (ref.[7]) present some modelling difficulties with respect to estimating the dilution of material taken up by the ambient flow directly over the horizontal source area. We have investigated approximating the sources by either line or point sources, placed at the source centre. Although such approximations are invalid over the source itself, they are found to lead to some improvement in agreement between model and data, especially at lower buoyancy fluxes.

Prior to lift-off, and possibly even after lift-off, our comparisons with the data of Hall and Walker indicate that a simple bulk Richardson number dependent entrainment model may adequately represent the enhanced rate of dilution of the plume. The data of Hall and Walker appear to support earlier work that the main parameter controlling buoyant plume lift-off is also bulk Richardson number, although whether this is also the case for non-buoyancy

conserving plumes remains open to question. It is hoped that the URAHFREP AHF field trials data may shed some further light on this aspect.

Critical lift-off Richardson numbers derived from correlations by Hall and Walker are given in this report. The data indicate that the concentration maximum starts to rise from the ground when  $Ri_* \sim 2$ . However, given the relative weakness of plume rise models in reproducing the observed suppression in lift-off for wide plumes, we suggest that it may be appropriate to keep the plume ground based up to at least  $Ri_* \sim 10$ , the point where ground level concentrations are about 10-20% maximum.

At higher buoyancies, when  $Ri_* > 10$ , one might wish to consider using a “free” plume model, such as the simple lift-off model discussed in this report, to predict plume rise and dilution. Such a model is found to be generally adequate for plume rise, except as discussed above for wide plumes, where compared with wind tunnel data plume rise appears to be overestimated. The plume rise from the widest source of Hall and Walker [7] indicate possible overestimation of plume rise height by a factor of approximately two. It is therefore suggested that in situations where wide buoyant ground based plumes may occur, and in the absence of improved lift-off models, suitable account should be taken of the possible overestimation by halving the predicted plume rise estimate, but keeping the plume concentration the same.

## 6 Acknowledgements

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