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The physics of heavy gas cloud dispersal

by

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SUMMARY

A review is presented of simple, bulk-property models of instantaneous heavy gas cloud releases. Using a common framework, analytic solutions to the models are derived for a class of adiabatic, and (in particular) isothermal flows. Not all of these solutions have been given previously. They allow a direct comparison of assorted air entrainment models and highlight the contrasting scaling properties inherent in each case.

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1. INTRODUCTION

Widespread storage and transport of toxic or inflammable heavy gases has led to considerable scientific interest in the way that heavy gas clouds disperse in the atmosphere. A gas cloud may be denser than air intrinsically (e.g. Cl_2) or because it is cold (e.g. CH_4 at its boiling point). Models for the dispersal of such gas clouds range from simple 'box' models, which we shall define and review here, to more complicated (but not necessarily more accurate) three dimensional codes.

We shall restrict our attention here to box models of instantaneous releases. There is a considerable number of such models available, and our primary aim is to achieve some form of clear comparison of the physical processes modelled and an understanding of why the results of these models are so diverse.

The models are not in general soluble analytically. We regard this as a major obstacle to attaining a physical understanding and so we shall focus entirely on buoyancy conserving flows where, as we shall show, explicit analytic solutions can be found. Buoyancy is conserved in two cases:

(a) Ambient temperature releases.

(b) Cold gas releases where all heating occurs by mixing with the atmosphere and where the molar specific heat of the contaminant is equal to that of the air.

In some models a further slight approximation is needed; we shall discuss this in detail below.

Case (a) embraces the experimental releases of Picknett (1978)[1] and the current Health and Safety Executive (HSE) trials at Thorney Island.

Case (b) involves neglecting ground heating.

All but the most recent models make the assumption that the cloud may be approximated by a 'slumping' phase followed (after an instantaneous transition) by a 'passive' phase. The differences between models are primarily in the treatment of gravity slumping and so we shall concentrate almost entirely on this aspect.

Our procedure (which follows that of Fay (1980)[2]) will be to construct a framework containing the common elements of the models, into which each model will fit naturally by specifying an entrainment model.

2. A GENERIC MODEL

2.1 Variables

We consider a cylindrical cloud of radius R, height h, volume V, uniform density ϱ and uniform temperature T. All these quantities are functions of time t. Other useful time dependent quantities are the concentration (mass of contaminant per unit volume) η, the mass fraction c, the mass of the cloud m, and the mass of the air in the cloud m_a. The total mass of contaminant in the cloud, m_g, is constant. The intrinsic density of the contaminant gas depends on

$$\Lambda \equiv 1 - \frac{M_a}{M_g} \qquad \ldots (2.1)$$

where M_a, M_g are the molecular weights of air and contaminant gas. For intrinsically heavy gases Λ is positive.

Taking the gas to be effectively incompressible (as is appropriate at low mach number) we write the equation of state as

$$(1 - \Lambda c)\,\varrho T = \varrho_a T_a \qquad \ldots (2.2)$$

where ϱ_a and T_a are the density and temperature of the ambient air.

We shall take independent variables to be R, V, and T. The dependent variables h, ϱ, η, c, m, and m_a are found as functions of these and the constants m_g, ϱ_a, T_a, Λ using the cylindrical geometry, the equation of state, and the definitions (below) of η, c, m, and m_a.

The results are:

$$h = \frac{V}{(\pi R^2)} \quad \text{... (2.3)}$$

$$\eta = \frac{m_g}{V} \quad \text{... (2.4)}$$

$$\varrho = \varrho_a \frac{T_a}{T} + \frac{m_g \Lambda}{V} \quad \text{... (2.5)}$$

$$c \equiv \frac{\eta}{\varrho} = \frac{m_g}{\varrho_a} \left[\frac{V T_a}{T} + \frac{m_g \Lambda}{\varrho_a} \right]^{-1} \quad \text{... (2.6)}$$

$$m \equiv \varrho V = \varrho_a V \frac{T_a}{T} + m_g \Lambda \quad \text{... (2.7)}$$

$$m_a \equiv m - m_g = \varrho_a V \frac{T_a}{T} + m_g (\Lambda - 1) \quad \text{... (2.8)}$$

We also define density variables

$$\Delta' = \frac{\varrho - \varrho_a}{\varrho_a} \quad \text{... (2.9)}$$

and

$$\Delta = \frac{\varrho - \varrho_a}{\varrho} \quad \text{... (2.10)}$$

and a buoyancy variable

$$b = \frac{1}{\pi} g V \Delta' \quad \text{... (2.11)}$$

where g is the acceleration due to gravity. In terms of the independent variables

$$b = \frac{1}{\pi} g \left[\left(\frac{T_a}{T} - 1 \right) V + \frac{m_g \Lambda}{\varrho_a} \right] \quad \text{... (2.12)}$$

For the isothermal case T is equal to T_a and hence (b) is conserved.

2.2 The heat capacity of the cloud and enthalpy conservation

The heat capacity of the cloud is

$$m c_p = m_g c_{pg} + m_a c_{pa}$$

which in terms of the independent variables becomes

$$m c_p = c_{pa} \left[\frac{\varrho_a T_a}{T} V - m_g \left(1 - \Lambda - \frac{c_{pg}}{c_{pa}} \right) \right] \quad \text{... (2.13)}$$

The enthalpy conservation law balances the enthalpy change in the cloud with that brought in by entrainment.

$$\frac{d}{dt}(mc_pT) = c_{pa}T_a\frac{dm_a}{dt} \qquad \text{... (2.14)}$$

which integrates to the conservation law

$$\left(\frac{T_a}{T} - 1\right)V + \frac{m_g}{\varrho_a}\left(1 - \Lambda - \frac{c_{pg}}{c_{pa}}\right)\frac{T}{T_a} = \text{constant} \qquad \text{... (2.15)}$$

This can be used to express T as a function of V leaving two independent variables, R and V. (If ground heating is important (2.14) no longer holds and so (2.15) is invalid – we shall discuss this later).

If the molar specific heats of the contaminant gas and air are identical (as predicted by the Kinetic Theory of Gases for molecules of the same structure) then

$$\frac{c_{pg}}{c_{pa}} = \frac{M_a}{M_g} = 1 - \Lambda \qquad \text{... (2.16)}$$

and (2.15) becomes

$$\left(\frac{T_a}{T} - 1\right)V = \text{constant} \qquad \text{... (2.17)}$$

and so (b) is again conserved. This buoyancy conservation simplifies the models considerably (see also Fay (1980),[2] Raj (1982)[3]) and so we shall look almost exclusively at this case.

2.3 Air entrainment

The above conservation law removes T from our list of independent variables leaving V and R. The evolution of the volume is modelled by

$$\frac{dV}{dt} = (\pi R^2)\,U_T + (2\pi Rh)\,U_E \qquad \text{... (2.18)}$$

where U_T and U_E are top, and edge entrainment velocities, which each model prescribes as function of the independent variables R and V. This prescription lies at the heart of the difference between models.

2.4 Gravity slumping

Coupled to (2.18) is the gravity front equation

$$\frac{dR}{dt} = U_f \qquad \text{... (2.19)}$$

where U_f is the front velocity, which again must be specified as a function of the independent variables. Models do not, however, differ widely in this; there are just two popular choices:

$$U_f = K\sqrt{gh\Delta'} \qquad \text{... (2.20a)}$$

and

$$U_f = K\sqrt{gh\Delta} \qquad \text{... (2.20b)}$$

where K is a constant. These do not differ by much unless the cloud is very dense. We shall adopt (2.20a) which can be derived most simply by balancing the height averaged pressure drop, $g(\varrho - \varrho_a)h/2$, across the front with a resistance pressure $(2K^2)^{-1}\varrho_a U_f^2$ from the ambient air. (The cloud is assumed to have reached its 'terminal' velocity). For those who prefer (2.20b) then (2.20a) may be regarded as an approximation – especially as much of the difference may be absorbed into a renormalisation of K.

In terms of the independent variables, we find

$$U_f = K\sqrt{b}\,R^{-1} \qquad \text{... (2.21a)}$$

or

$$U_f = K\sqrt{b}\,R^{-1}\sqrt{\frac{V}{V + \frac{b\pi}{g}}} \qquad \text{... (2.21b)}$$

The merits of our choice (a) are manifest. Conservation of buoyancy allows one to integrate (2.19) immediately. Before doing this, however, we shall write the resultant equations (2.18, 2.19 and 2.21a) in dimensionless form.

2.5 Dimensionless equations

We scale R, V and h by their initial values R_0, V_0 and h_0. The time scale is defined in terms of the initial slumping velocity to be

$$t_0 = \frac{R_0}{2\,U_f\,(o)} = \frac{R_0^2}{2K\sqrt{b}} \qquad \text{... (2.22)}$$

The dimensionless variables are then

$$\left.\begin{aligned} \hat{t} &= \frac{t}{t_0} \\ \hat{r} &= \frac{R}{R_0} \\ \hat{v} &= \frac{V}{V_0} \\ \hat{h} &= \frac{h}{h_0} \end{aligned}\right\} \qquad \text{... (2.23)}$$

and the equations governing the behaviour of the cloud are

$$\frac{d\hat{r}}{d\hat{t}} = \frac{1}{2\hat{r}} \qquad \text{... (2.24)}$$

and

$$\frac{d\hat{v}}{d\hat{t}} = \hat{r}^2\left[\frac{U_T\,t_0}{h_0}\right] + \frac{2\hat{v}}{\hat{r}}\left[\frac{U_E\,t_0}{R_0}\right] \qquad \text{... (2.25)}$$

These can be solved most conveniently as follows. Firstly (2.24) integrates to

$$\hat{r} = \sqrt{1 + \hat{t}} \qquad \text{... (2.26)}$$

Secondly $\hat{v}$ is found as a function of $\hat{r}$ by dividing (2.25) by (2.24) giving

$$\frac{d\hat{v}}{d\hat{r}} = 2\alpha\hat{v}\left[\frac{U_E}{U_E\,(o)}\right] + 2\beta\hat{r}^3\left[\frac{U_T}{U_T\,(o)}\right] \qquad \text{... (2.27)}$$

where the effective edge and top entrainment coefficients are given by

$$\alpha = \frac{U_E(o)}{U_f(o)} = \frac{2t_o U_E(o)}{R_o} \qquad \text{... (2.28)}$$

and

$$\beta = \frac{1}{2}\left[\frac{U_T(o)}{U_f(o)}\right]\left[\frac{R_o}{h_o}\right] = \frac{t_o U_T(o)}{h_o} \qquad \text{... (2.29)}$$

2.6 Analytic models

To complete the specification of the model we need the entrainment velocities U_T and U_E as a function of the state of the cloud and some constants. α and β are then easily calculated in terms of these constants and the initial cloud parameters. In terms of the independent dimensionless variables we write

$$\frac{U_E}{U_E(o)} = \hat{u}_E(\hat{r}, \hat{v}) \qquad \text{... (2.30)}$$

$$\frac{U_T}{U_T(o)} = \hat{u}_T(\hat{r}, \hat{v}) \qquad \text{... (2.31)}$$

and thus to each model there will be a corresponding characteristic ordinary differential equation (2.27) for $\hat{v}(\hat{r})$. This may or may not be soluble analytically. Fortunately in a wide set of models it is. We shall examine just two possible classes of model here.

Class I:

$\hat{u}_E(\hat{r})$ and $\hat{u}_T(\hat{r})$ are independent of $\hat{v}$.

In this case (2.27) is linear and the solution is

$$\hat{v}(\hat{r}) = F_\alpha(\hat{r}) + 2\beta \int_1^{\hat{r}} dw \left[\frac{F_\alpha(\hat{r})}{F_\alpha(w)}\right] w^3 \hat{u}_T(w) \qquad \text{... (2.32)}$$

where

$$F_\alpha(\hat{r}) = \exp\left(2\alpha \int_1^{\hat{r}} dw\, \hat{u}_E(w)\right) \qquad \text{... (2.33)}$$

Class II:

$\hat{u}_E$ is independent of $\hat{v}$ and $\hat{u}_T$ takes the form

$$\hat{u}_T = \hat{v}^{1-\mu} f_T(\hat{r}) \qquad \text{... (2.34)}$$

where μ is a constant. In this case (2.27) becomes

$$\frac{d\hat{v}}{d\hat{r}} = \hat{v}\,[2\alpha\,\hat{u}_E(\hat{r})] + [2\beta\hat{r}^3 f_T(\hat{r})]\,\hat{v}^{1-\mu} \qquad \text{... (2.35)}$$

This is a non-linear equation for $\hat{v}(\hat{r})$ but the general solution can be found by noticing that it can be written in the form

$$\frac{d}{d\hat{r}}(\hat{v}^\mu) = [2\mu\alpha\,\hat{u}_E(\hat{r})]\,\hat{v}^\mu + [2\mu\beta\,\hat{r}^3 f_T(\hat{r})] \qquad \text{... (2.36)}$$

whence

$$\hat{v} = \left\{ F_{\mu\alpha}(\hat{r}) + 2\mu\beta \int_1^{\hat{r}} dw \left[\frac{F_{\mu\alpha}(\hat{r})}{F_{\mu\alpha}(w)} \right] w^3 f_T(w) \right\}^{1/\mu} \quad \ldots (2.37)$$

where $F_{\mu\alpha}$ is obtained from (2.33) by replacing α by $\mu\alpha$.

We note that Class I is actually a subset of Class II with $\mu = 1$, but we regard the two classes as sufficiently different physically and we shall distinguish them throughout. There may be more general analytically soluble models but we have not sought them because, at least for the isothermal case, with the proviso that (2.20a) is taken as the slumping formula, ALL the simple box models which we have found fall into one or other of these classes.

This Section comprises essentially a 'do-it-yourself heavy gas model kit': one merely has to plug in the desired entrainment model for U_E and U_T and, provided the non-dimensional equivalents fall into Classes I or II, the answer may be read off from (2.32) or (2.37). In the following Section we shall use these results to give solutions to assorted heavy gas slumping models. More detailed discussion is contained in Section 4.

3. MODELS

To compare the various models we shall give the results in the dimensionless form given in the previous Section. We shall use $\hat{r}$ as the independent variable, which is related to time through (2.26). In each case we shall give the height function.

$$\hat{h}(\hat{r}) = \frac{\hat{v}}{\hat{r}^2} \quad \ldots (3.1)$$

Other variables such as density and concentration are simple functions of $\hat{v}$

$$\Delta' = \frac{\Delta_0'}{\hat{v}} \quad \ldots (3.2)$$

$$\eta = \frac{\eta_0}{\hat{v}} \quad \ldots (3.3)$$

where Δ_0' and η_0 are the initial values.

3.1 The model of Van Ulden (1974)

Van Ulden (1974)[4] uses the slumping formula (2.20b). Our approach (2.20a) must be thought of as an approximation to this. In this model there is no top entrainment and the edge entrainment is taken as proportional to the front velocity:

$$U_T = 0 \quad \ldots (3.4)$$

$$U_E = \alpha U_f \quad \ldots (3.5)$$

Thus

$$\hat{u}_T = 0 \quad \ldots (3.6)$$

$$\hat{u}_E = \frac{1}{\hat{r}} \quad \ldots (3.7)$$

(using 2.20a; 2.21a), and so this model falls into the first of the two classes. Equations (2.32), (2.33) give

$$\hat{v}(\hat{r}) = F_\alpha(\hat{r}) = \hat{r}^{2\alpha} \quad \ldots (3.8)$$

and hence

$$\hat{h} = \hat{r}^{-2(1-\alpha)} \qquad \text{... (3.9)}$$

From his experiment, Van Ulden was led to believe that α was close to zero. For this case his slumping formula (2.20b) yields an exact solution of the form (2.26) except that the time scale t_0 in (2.23) must be replaced by $t_0 \sqrt{\varrho_o/\varrho_a}$. (This effectively means a reformalisation of K). For non-zero entrainment the situation is not so simple but we regard this evidence as encouraging the belief that the choice between (2.20a) and (2.20b) is not crucial.

Van Ulden considers the slumping phase to terminate when ½ dR/dt decreases to the friction velocity u_*. This is when

$$\hat{r} = \frac{R_o}{4u_* t_o} \qquad \text{... (3.10)}$$

3.2 The model of Germeles and Drake (1975)

By contrast with Van Ulden, Germeles and Drake (1975)[5] consider the edge entrainment to be zero and top entrainment to be proportional to the front velocity, for which they use equation (2.20a). That is

$$U_E = 0 \qquad \text{... (3.11)}$$

$$U_T = \frac{2}{3} \alpha_{G.D.} U_f \qquad \text{... (3.12)}$$

and so

$$\hat{u}_E = 0 \qquad \text{... (3.13)}$$

$$\hat{u}_T = \frac{1}{\hat{r}} \qquad \text{... (3.14)}$$

and

$$\beta = \frac{1}{3} \alpha_{G.D.} \left[\frac{R_o}{h_o}\right] \qquad \text{... (3.15)}$$

The model is again in Class I of the previous section with

$$F_\alpha (\hat{r}) = 1 \qquad \text{... (3.16)}$$

$$\hat{v} (\hat{r}) = 1 + \frac{2}{3} \beta (\hat{r}^3 - 1) \qquad \text{... (3.17)}$$

from (2.32) and (2.33). The height function is

$$\hat{h} (\hat{r}) = \left(1 - \frac{2}{3}\beta\right) \hat{r}^{-2} + \frac{2}{3} \beta \hat{r} \qquad \text{... (3.18)}$$

This model considers the slumping phase to terminate when U_f is equal to the wind velocity u_w. That is when

$$\hat{r} = \frac{R_o}{2t_o u_w} \qquad \text{... (3.19)}$$

3.3 The model of Fay (1980)[2]

In reviewing different models Fay (1980) considers a model having both top and edge entrainment velocities proportional to U_f. That is

$$\hat{u}_E = \hat{u}_T = \frac{1}{\hat{r}} \qquad \ldots (3.20)$$

This again fits Class I of our scheme with

$$F_\alpha(\hat{r}) = \hat{r}^{2\alpha} \qquad \ldots (3.21)$$

and

$$\hat{u} = (1 - \gamma)\,\hat{r}^{2\alpha} + \gamma\,\hat{r}^3 \qquad \ldots (3.22)$$

$$\hat{h} = (1 - \gamma)\,\hat{r}^{-2(1-\alpha)} + \gamma\,\hat{r} \qquad \ldots (3.23)$$

with

$$\gamma = \frac{2\beta}{3 - 2\alpha} \qquad \ldots (3.24)$$

where β takes a form similar to (3.15). This model is very close to that of Germeles and Drake for small edge entrainment coefficient α.

3.4 The model of Fryer and Kaiser (1979)[6]

This model incorporates edge entrainment in the same way as those above, and uses the slumping velocity (2.20a). The top entrainment velocity is modelled in the form

$$U_T = \frac{\alpha' u_1}{Ri} \qquad \ldots (3.25)$$

where u_1 is a longitudinal turbulence velocity. The Richardson number Ri is defined as

$$Ri = \frac{g\,\Delta'\,\ell}{u_1^2} \qquad \ldots (3.26)$$

where the turbulence length scale ℓ is taken from Taylor et al. (1970)[7] to be

$$\ell = h_r \left(\frac{h}{h_r}\right)^{\mu} \qquad \ldots (3.27)$$

with a reference height $h_r = 30 \cdot 2$ m and $\mu = 0 \cdot 48$. Thus the entrainment model is

$$U_E = \alpha\, U_f \qquad \ldots (3.28)$$

$$U_T = \frac{\alpha' u_1^3}{\pi\, b\, h_r} \left(\frac{h}{h_r}\right)^{\mu} V \qquad \ldots (3.29)$$

or

$$\hat{u}_E = \frac{1}{\hat{r}} \qquad \ldots (3.30)$$

$$\hat{u}_T = \hat{v}^{1-\mu}\, \hat{r}^{2\mu} \qquad \ldots (3.31)$$

with

$$\beta = \frac{\pi \alpha' u_1^3}{2K b^{3/2}} (\pi h_r)^{\mu - 1} V_0^{-\mu} R_0^{4+2\mu} \qquad \text{... (3.32)}$$

The constant β depends on weather conditions through u_1, which is defined as a multiple of u_* according to the weather category by

Weather	A,B	C,D	E,F
u_1	$3 \cdot 0u_*$	$2 \cdot 4u_*$	$1 \cdot 6u_*$

... (3.33)

Equations (3.30) and (3.31) place this model in the second class discussed in Section 2 and so we have

$$F_{\mu\alpha}(\hat{r}) = \hat{r}^{2\mu\alpha} \qquad \text{... (3.34)}$$

and

$$\hat{v} = \left\{(1 - \gamma)\, \hat{r}^{2\mu\alpha} + \gamma\, \hat{r}^{2(2+\mu)}\right\}^{1/\mu} \qquad \text{... (3.35)}$$

or

$$\hat{h} = \left\{(1 - \gamma)\, \hat{r}^{-2\mu(1-\alpha)} + \gamma\, \hat{r}^{4}\right\}^{1/\mu} \qquad \text{... (3.36)}$$

with

$$\gamma = \frac{\beta\mu}{2 + \mu(1 - \alpha)} \qquad \text{... (3.37)}$$

The condition for the cloud to become passive is more complicated in this model as there are a number of criteria. If both of the conditions

$$U_f < 2 \cdot 14\, u_w \left(\frac{h}{2}\right) \frac{d\sigma_y}{dx} \qquad \text{... (3.38)}$$

and

$$\frac{\alpha'}{Ri} > 1 \qquad \text{... (3.39)}$$

hold then the cloud is a considered passive. Irrespective of this if

$$\Delta' < 8 \cdot 16\ 10^{-4} \qquad \text{... (3.40)}$$

the cloud is considered passive. ($\sigma_y(x)$ is the usual phenomenological gaussian plume function for passive dispersion and $u_w(z)$ is the wind velocity as a function of height).

3.5 The model of Cox and Carpenter (1979)[8]

For the purposes of this study, the model of Cox and Carpenter (1979) does not differ significantly from that of Fryer and Kaiser (and both owe much to that of Cox and Roe (1977)).[9] Most of the differences of detail in these models (such as phase changes in the cloud and effects of water vapour) are outside the scope of this review. We note, however, that the passive transition criterion is simpler

$$U_f = u_w \frac{d\sigma_y}{dx} \qquad \text{... (3.41)}$$

– c.f. (3.38).

3.6 The model of Picknett (1978)[1]

In the Appendix to the report on the 1978 Porton Down experiments, Picknett (1978), presents the following analytic model.

The entrainment velocities are

$$U_E = \alpha U_f \qquad \text{... (3.42)}$$

$$U_T = \frac{\beta_p u_*}{Ri} \qquad \text{... (3.43)}$$

where β_p is a dimensionless, constant coefficient and the Richardson number is defined as

$$Ri = \frac{g h \Delta'}{u_*^2} \qquad \text{... (3.44)}$$

Comparing with Fryer and Kaiser[6] (Sub-section 3.4) we see that the models are the same except that here u_* replaces the turbulance velocity u_1, and the Richardson number goes at height rather than $h^{0.48}$. Thus

$$\hat{u}_E = \frac{1}{\hat{r}} \qquad \text{... (3.45)}$$

$$\hat{u}_T = \hat{r}^2 \qquad \text{... (3.46)}$$

$$\beta = \frac{\pi \beta_p u_*^3}{2 K b^{3/2}} \cdot \frac{R_o^6}{V_o} \qquad \text{... (3.47)}$$

and

$$\hat{u} = (1 - \gamma)\, \hat{r}^{2\alpha} + \gamma \hat{r}^6 \qquad \text{... (3.48)}$$

$$\hat{h} = (1 - \gamma)\, \hat{r}^{-2(1-\alpha)} + \gamma \hat{r}^4 \qquad \text{... (3.49)}$$

where

$$\gamma = \frac{\beta}{3 - \alpha} \qquad \text{... (3.50)}$$

The criterion for passive dispersion demands

$$\sqrt{2 g \Delta' h} \leqslant 3 \cdot 75 u_* \qquad \text{... (3.51)}$$

so that the transition takes place when

$$\hat{r} = \frac{\sqrt{2b}}{3 \cdot 75 u_* R_o} \qquad \text{... (3.52)}$$

3.7 The model of Eidsvik (1980)[10]

The model of Eidsvik differs from all the above in a number of ways. Turbulence due to convection caused by the temperature difference between the cloud and the ground is considered to be of major importance. We thus regard the neglect of all ground heating effects as a more serious omission in the context of this model. In this case, then, the following is considered to apply only to ambient temperature releases.

The edge entrainment velocity is taken to be

$$U_E = \frac{\alpha U_f^2}{U_f(o)} \quad \ldots (3.53)$$

This form is entirely ad hoc and is an attempt to reconcile the model with the data of Van Ulden (1974)[4] as well as with that of Picknett (1978)[1] – something which previous models had been unable to do.

The top entrainment velocity is modelled in terms of a turbulence velocity v_E as

$$U_T = \frac{\alpha_4}{(\alpha_6^* + Ri)} v_E \quad \ldots (3.54)$$

where α_4 and α_6^* are constants and the Richardson number Ri is defined by

$$Ri = \frac{g h \Delta}{v_E^2} \quad \ldots (3.55)$$

Note that this goes linearly with the height (c.f. Picknett). The turbulence velocity is defined by

$$v_E = \alpha_3 \sqrt{\frac{c_f}{2}} \sqrt{\frac{4}{9} U_f^2 + u_w^2} \quad \ldots (3.56)$$

for ambient temperature releases, where α_3 and c_f are constants and u_w is the wind speed. The turbulence velocity is thus determined both by ambient atmospheric turbulence (u_w) and mechanical turbulence caused by the cloud (U_f).

Note that Eidsvik takes Δ rather than Δ' both in the slumping formula and in the Richardson number (3.55). To make further analytic progress we shall use Δ' in both places. This should not change the essential features of the model.

To express the entrainment velocity concisely we define the following dimensionless groups

$$\xi = \frac{2}{3} \frac{U_f(o)}{u_w} = \frac{2 K \sqrt{b}}{3 R_o u_w} \quad \ldots (3.57)$$

$$\chi^2 = \frac{9}{4K^2} \left(\frac{\alpha_6^* \alpha_3^2 c_f}{2}\right)^{-1} \quad \ldots (3.58)$$

and

$$\beta = \left[\frac{R_0}{h_0}\right] \frac{\alpha_4 \alpha_3}{3 \alpha_6^*} \sqrt{\frac{c_f}{2}} \frac{(1 + \xi^{-2})^{3/2}}{(1 + \chi^2 + \xi^{-2})} \quad \ldots (3.59)$$

The entrainment velocities are then

$$\hat{u}_E(\hat{r}) = \frac{1}{\hat{r}^2} \quad \ldots (3.60)$$

$$\hat{u}_T(\hat{r}) = \left[\frac{1 + \chi^2 + \xi^{-2}}{(1 + \xi^{-2})^{3/2}}\right] \frac{\left(1 + \frac{\hat{r}^2}{\xi^2}\right)^{3/2}}{\hat{r}\left(1 + \chi^2 + \frac{\hat{r}^2}{\xi^2}\right)} \quad \ldots (3.61)$$

From (3.57) we see that the parameters ξ measures the relative strengths of mechanical and atmospheric turbulence. The parameter χ measures the strength of the density interface suppression, for which the relevant quantity is

$$\frac{Ri}{\alpha_6^*} = \frac{\chi^2}{\left(1 + \frac{\hat{r}^2}{\xi^2}\right)} \qquad \text{... (3.62)}$$

Equations (3.60, 3.61) put this model in Class I defined in Section 2 whence we have

$$F_\alpha(\hat{r}) = \exp\left(2\alpha\left(1 - \frac{1}{\hat{r}}\right)\right) \qquad \text{... (3.63)}$$

and

$$\hat{v}(\hat{r}) = e^{-2\alpha\left(1-\frac{1}{\hat{r}}\right)} + 2\beta\int_1^{\hat{r}} dw\, e^{\left(\frac{2\alpha}{w}-\frac{2\alpha}{\hat{r}}\right)} w^3\, \hat{u}_T(w) \qquad \text{... (3.64)}$$

Owing to the form of (3.61) this integral must be done numerically.

Eidsvik's model is formulated to be valid asymptotically as $t \to \infty$ ($\hat{r} \to \infty$) without the need for a sudden transition to passive behaviour. The asymptotic passive behaviour is

$$\hat{v} \underset{\hat{r}\to\infty}{\simeq} \left[\beta(1 + \xi^2)^{-3/2}\,(1 + \chi^2 + \xi^{-2})\,\xi^{-1}\right] \hat{r}^4 \qquad \text{... (3.65)}$$

for non-zero wind, and in zero wind conditions ($\xi \to \infty$) it is

$$\hat{v} \underset{\hat{r}\to\infty}{\simeq} \frac{2}{3}\beta\,\hat{r}^3 \qquad \text{... (3.66)}$$

3.8 The model of Fay and Ranck (1981)[11]

Fay and Ranck (1981) consider an analytic model with no edge entrainment ($U_E = 0$) but which is otherwise similar in spirit to that of Eidsvik. The model is designed to have no sudden transition to passive behaviour. In this case the top entrainment velocity is

$$U_T = \frac{c_1\, c_2\, u_*}{\sqrt{c_2^2 + c_1^2\, Ri^2}} \qquad \text{... (3.67)}$$

where c_1, and c_2 are constants and the Richardson number is defined as

$$Ri = \frac{g\, h\, \Delta'}{u_*^2} \qquad \text{... (3.68)}$$

Note that Δ' is preferred here over Δ, and also in the slumping formula.

We define the dimensionless constant

$$\phi = \sqrt{\frac{c_1}{c_2}\, Ri(o)} = \sqrt{\frac{c_1}{c_2}\,\frac{b}{u_*^2\, R_0^2}} \qquad \text{... (3.69)}$$

so that

$$\hat{u}_T = \sqrt{1 + \phi^4}\left(\frac{\hat{r}^2}{\sqrt{\hat{r}^4 + \phi^4}}\right) \qquad \text{... (3.70)}$$

with

$$\beta = \frac{c_1}{2K\sqrt{1 + \phi^4}}\left[\frac{u_*\, R_o}{\sqrt{b}}\right]\left[\frac{R_o}{h_o}\right] \qquad \text{... (3.71)}$$

and

$$\frac{c_1}{c_2} \, Ri = \frac{\phi^2}{\hat{r}^2} \qquad \text{... (3.72)}$$

This model again falls into Class I defined in Section 2 with the solution

$$F_\alpha(\hat{r}) = 1 \qquad \text{... (3.73)}$$

owing to the absence of top entrainment and

$$\hat{v} = 1 + \gamma \left[\hat{r}^2 \sqrt{\hat{r}^4 + \phi^4} - \sqrt{1 + \phi^4} - \phi^4 \, \ell n \left(\frac{\hat{r}^2 + \sqrt{\hat{r}^4 + \phi^4}}{1 + \sqrt{1 + \phi^4}} \right) \right] \qquad \text{... (3.74)}$$

where

$$\gamma = \frac{\beta}{2} \sqrt{1 + \phi^4} \qquad \text{... (3.75)}$$

Asymptotically we have

$$\hat{v} \underset{\hat{r} \to \infty}{\simeq} \gamma \, \hat{r}^4 \qquad \text{... (3.76)}$$

in the same way as in Eidsvik's model. (Equation 3.65).

In zero wind conditions ($u_* = 0$) there is no entrainment giving a constant volume.

3.9 The downwind progress of the cloud

All the above models assume that the cloud moves downwind with the speed of the wind. Thus the downwind distance is given by

$$x = \int_o^t u_w \, dt \qquad \text{... (3.77)}$$

Some models assume a constant wind speed so that

$$x = u_w \, t \qquad \text{... (3.78)}$$

but others take a given wind profile and use the value at the half height of the cloud as the velocity u_w. In this case

$$x = R_0 \int_1^{\hat{r}} \frac{u_w}{U_f(o)} \, \hat{r} \, d\hat{r} \qquad \text{... (3.79)}$$

where we have gone back to using $\hat{r}$ as the time evolution parameter. This integral is not in general doable analytically. One exception to this is the model of Van Ulden (1974)[4] with a logarithmic wind profile

$$u_w(z) = u_w(10) \, \frac{\ell n \left(\frac{z}{z_0} \right)}{\ell n \left(\frac{10}{z_0} \right)} \qquad \text{... (3.80)}$$

which gives

$$\frac{x}{R_0} = \left[\frac{u_w \left(\frac{h_0}{2} \right) t_o}{R_0} \right] \left[(\hat{r}^2 - 1) - \frac{(1 - \alpha)}{\ell n \left(\frac{h_0}{2 z_0} \right)} \left\{ 1 + \hat{r}^2 \ell n \, \hat{r}^2 - \hat{r}^2 \right\} \right] \qquad \text{... (3.81)}$$

The first term is just what one would obtain with a fixed wind speed of u_w $(h_0/2)$.

3.10 The passive dispersion phase

All but the last two of the models considered have a sudden transition to a passive phase which is based on the standard Gaussion plume approach. The main differences must come, therefore, from the slumping phase, and in particular from the entrainment models. We shall not discuss the passive phase in detail here, but rather concentrate on the slumping phase as that which provides a ''source term'' for passive dispersion.

4. A COMPARISON OF MODELS

4.1 Common features

The slumping formula is common to all the models – we do not regard the choice between Δ' and Δ ((2.20a) or (2.20b)) as a significant difference.

The edge entrainment mechanism (in those models which have it) is always one of mechanical turbulence, being dependent on U_f. Its effect is always most crucial in the initial stages of the slumping process.

4.2 Top entrainment

The models can be classified by their top entrainment mechanism as follows:

No top entrainment:	Van Ulden (1974)[4]
Top entrainment due to mechanical turbulence:	Germeles and Drake (1975)[5] Fay (1980)[2]
Top entrainment due to ambient atmospheric turbulence:	Picknett (1978)[1] Cox and Carpenter (1979)[8] Fryer and Kaiser (1979)[6] Fay and Ranck (1981)[11]
Top entrainment due to both mechanical and ambient turbulence:	Eidsvik (1980)[10]

Each class has its own distinctive features. Fay (1982)[12] has drawn attention to the fact that the models including top entrainment due to mechanical turbulence give an increasing cloud height even in windless conditions, violating energy conservation. We see clearly how this comes about from Equation (2.32). Neglecting edge entrainment (i.e. taking $F_\alpha(\hat{r}) = 1$) we see that $\hat{v} \gtrsim \hat{r}^2$ if $\hat{u}_T \gtrsim \hat{r}^{-2}$. In these circumstances the height will increase with time. If $U_T \sim U_f \sim \hat{r}^{-1}$ these conditions are clearly fulfilled and energy conservation is violated. (This is not the case for the ambient turbulence models where $\hat{u}_T$ vanishes in the limit of zero wind). It may be, then, that the mechanical turbulence models over-estimate entrainment even in relatively windy conditions.

Some further comments are in order on the top entrainment in the model of Eidsvik (1980).[10] In this model the two turbulent mechanisms are both present. The most interesting situation is when $\xi \gg 1$: a high, dense cloud in a low wind. In this case there is a region ($\hat{r} \ll \xi$) where mechanical turbulence dominates and another ($\hat{r} \gg \xi$) where ambient atmospheric turbulence dominates. This situation, though, may be rare. In zero wind the first mechanism is the only one. For the more usual releases $\xi = O(1)$ (c.f. Van Ulden's release where $\xi \sim 0 \cdot 9$) and the ambient turbulence dominates the entrainment for most of the expansion.

4.3 Transition criteria

All the models compare the front velocity U_f with a velocity characteristic of the ambient, atmospheric flow to establish a transition criterion for passive behaviour. This is true even of the more sophisticated models where the transition is gradual. This 'atmospheric' velocity is either the wind speed u_w or the friction velocity u_*.

The models which take u_w tend to have a much earlier transition than the others. For example, in the model of Germeles and Drake (1975)[5] the slumping barely begins before the transition to passive behaviour is assumed. Thus in this model the height increase in the context of gravity spreading will only be seen in very low or zero wind conditions – a distinctly odd result as we have remarked above. The model of Eidsvik (1980)[10] also uses U_f/u_w to define the transition criterion (at $\hat{r} \approx \xi \sqrt{\chi^2 - 1}$, see Equation (3.62)).

All the other models compare U_f with u_* to establish the transition criterion. This is also true in the smooth transition of the model of Fay and Ranck (1981)[11] which occurs around $\hat{r} \sim \phi \approx U_f/u_*$ – (see Equation (3.72)).

We regard the last two models considered in Section 3, with their smooth transition to passive behaviour as a significant advance on earlier models. However, we note that a choice is still made as to when the asymptotic, 'passive' behaviour set in. This is clear when one compares the two models.

4.4 Turbulence suppression

Van Ulden's model[4] with no top entrainment, if one extrapolates the gravity spreading phase, predicts

$$\hat{h} \sim \hat{r}^{-2(1-\alpha)} \qquad \dots (4.1)$$

The models with mechanical top entrainment without the Richardson number, density interface suppression of turbulence give

$$\hat{h} \sim \hat{r} \qquad \dots (4.2)$$

Those with turbulence suppression give

$$\hat{h} \sim \hat{r}^4 \qquad \text{(Picknett)} \qquad \dots (4.3)$$

or

$$\hat{h} \sim \hat{r}^{8.33} \qquad \text{(Fryer and Kaiser/ Cox and Carpenter)}^{(6,8)} \qquad \dots (4.4)$$

(All these models are truncated in favour of a passive model before this asymptotic behaviour sets in!) These more rapid increases are to do with the dependence of the Richardson number on height as well as on density. The Richardson number decreases as the density of the cloud decreases giving an increase in the entrainment rate. Partly counteracting this effect is the tendency of the Richardson number to increase with the height of the cloud. This latter effect is less prominent in the models of Fryer and Kaiser and of Cox and Carpenter where the behaviour is $h^{0.48}$ instead of h^1. Thus, in these models the density decrease does more to increase the entrainment rate as time goes on, explaining (4.4). The dependence of the turbulence suppression factor on height is thus crucial to the entrainment mechanism, a feature which has been little discussed in the past. This dependence also affects the scaling properties of the model significantly as we shall see in Section 5.

5. SCALING

5.1 The time scale

The slumping time scale is given in (2.22) and is

$$t_0 = \frac{1}{2K\sqrt{g}} (\Delta_0' h_0)^{-\frac{1}{2}} R_0 \qquad \text{... (5.1)}$$

The slumping velocity scale is

$$U_f(o) = \frac{R_0}{2t_0} = K\sqrt{g}\,(\Delta_0' h_0)^{\frac{1}{2}} \qquad \text{... (5.2)}$$

The time scale decreases with increasing density as expected. There are numerous ways to express the variation with the cloud's initial dimensions in terms of height, radius, volume, aspect ratio, etc. Interestingly t_0 increases as the height and radius are increased in proportion, as also does $U_f(o)$; increasing the scale in this straightforward way means faster slumping for a longer time. This emphasises the importance of mechanical turbulence in large scale releases, and, in view of the possibility of large scale accidental releases, motivates further study of the slumping phenomenon.

5.2 Entrainment

Let us consider how the results of the various models of entrainment vary with the initial size and density of the cloud. In all the models considered except those of Eidsvik (1980)[10] and of Fay and Ranck (1981)[11] the non-dimensional height variable $\hat{h}$ behaves as

$$\hat{h} = [(1-\gamma)\,\hat{r}^{-a} + \gamma\,\hat{r}^{b}]^{1/c} \qquad \text{... (5.3)}$$

Of the constants a, b, c and γ, only γ depends on the initial release values $R_0\ h_0\ \varrho_0$. Values of a, b and c and the behaviour of γ in terms of R_0, h_0 and Δ_0' are given below for the various models

Model	a	b	c	$\gamma \sim$
Van Ulden (1974)	$2(1-\alpha)$	-	1	$= 0$
Germeles & Drake (1975)	2	1	1	$R_0\,h_0^{-1}$
Fay (1980)	$2(1-\alpha)$	1	1	$R_0\,h_0^{-1}$
Fryer & Kaiser (1979) / Cox & Carpenter (1979) *	$2\mu(1-\alpha)$	4	μ	$\Delta_0'^{-3/2}\,R_0\,h_0^{-(\mu+3/2)}$
Picknett (1978)	$2(1-\alpha)$	4	1	$\Delta_0'^{-3/2}\,R_0\,h_0^{-5/2}$

* $\mu = 0{\cdot}48$ in these models

Thus these models scale ($\hat{h}(\hat{r})$ and hence $\hat{h}(\hat{t})$, $\hat{v}(\hat{t})$ etc. are invariant) if the initial conditions R_0, h_0, ϱ_0 are varied such that the groups in the right hand column do not change. This scaling, of course, only applies to the slumping phase.

The models of Eidsvik (1980)[10] and of Fay and Ranck (1981)[11] are more complicated. In Eidsvik's model two constants have scaling behaviour. These are $\beta\,(R_0/h_0,\ \Delta_0' h_0)$ and $\xi\,(\Delta_0' h_0)$ defined in (3.57) and (3.59). The same is true in the model of Fay and Ranck where the relevant constants are $\beta\,(R_0/h_0,\ \Delta_0' h_0)$ and $\phi\,(\Delta_0' h_0)$ defined in (3.69) and (3.71). Thus both these models predict that R_0, h_0 and ϱ_0 may be varied such that both R_0/h_0 and $\Delta_0' h_0$ are constant and the physics will be scale invariant. There is one fewer degree of freedom here but the scaling is appropriate to the whole of the expansion – not just where gravity effects dominate.

In these two models the initial aspect ratio R_0/h_0 and the initial front velocity and Richardson number must be kept constant. The tabulated models either scale simply with the aspect ratio, or with $[R_0/h_0]\ [Ri(o)^{-1}]\ [U_f(o)^{-1}]$. The differences between Picknett[1] and Fryer and Kaiser[6] in this "aspect ratio – Richardson number – front velocity" scaling law lie in the definition of the Richardson number.

Finally let us remark that the scaling laws can be generalised to allow for alterations in the wind speed (or friction velocity). The relevant factors are easily found from the expression for γ for the various models.

5.3 Testing model scaling laws

There is some evidence that the area A of the cloud does increase linearly with time as predicted by (2.26). (Van Ulden (1974),[4] Picknett (1978)).[1] This is particularly good in the case where a cloud was released in calm atmospheric conditions. In Figure 1 we have shown the best slope dA/dt against $\sqrt{b}$ for each experiment of Picknett and that of Van Ulden. The expected linear behaviour implicit in (2.21a) is also shown. For Van Ulden's experiment K = 0·86 is the best value. For Picknett's experiment K = 1·0 is on average better. We do not regard this discrepancy as large (given the scatter on the data) and await with interest the outcome of the HSE Thorney Island experiments which will have $\sqrt{b} \sim 80\,m^2 s^{-1}$.

The scaling of $\hat{h}(\hat{r})$ is perhaps harder to measure. Here the interest lies in the fact that it distinguishes between models. We emphasise that it distinguishes between the physical assumptions of the models not merely between different values of free parameters. An experiment with a good range of R_0, h_0 and Δ_0' (and fixed weather conditions!!) would be of enormous value in testing these models. These conditions would seem to point to wind tunnel tests as the most practical approach. Note that to test Eidsvik's model, or that of Fay and Ranck, all three of R_0, h_0 and ϱ_0 must be varied.

The importance of the scaling law can be seen by considering releases with a given aspect ratio but different initial heights. In this case γ is constant, varies as $\Delta_0'^{-3/2}\, h_0^{-0.98}$ or as $\Delta_0'^{-3/2}\, h_0^{-3/2}$ depending on which of the tabulated models one accepts. Between wind tunnel experiments and field tests γ can be invariant or vary through a factor 10^3 or so depending on the model.

6. DISCUSSION AND SUMMARY

6.1 Cold clouds

We regard most of this review as being on relatively safe ground for ambient temperature releases. For cold clouds the effects of specific heats (i.e. a ratio other than that in Equation (2.16)) and of ground heating should be considered. Experimentally the conservation law (2.15) could, in principle, be tested. The size of any violation would indicate the importance of ground heating (or other non-entrainment heating) effects. The effect of such heating in the models can of course be evaluated by implementation of the respective computer codes.* The effect of different molar specific heats and of ground heating on the scaling laws would also be of considerable interest. The only analytic progress in these directions, which we can envisage is to do an expansion in a small parameter $(1 - \Lambda - c_{pg}/c_{pa})$ for the specific heat problem.

Heating from the ground at temperature T_g involves introducing a term $\pi R^2 f (T_g - T)$ to the right hand side of (2.14) and mathematically changes the entire character of the problem. Analytic progress looks unlikely here.

6.2 Summary and conclusions

We have reviewed simple box models of the dispersion of an 'instantaneously' released heavy gas cloud. The emphasis has been firmly on the physics of the entrainment process during gravity spreading. The models were presented in the context of a unified framework and analytic solutions to each were developed. Some, but not all, of these have already been given in the literature. The

**Indications, in the model of Fryer and Kaiser (1979),[6] are that the effect is small in general.*

different physical assumptions in the models give rise to very diverse predictions. These are not merely due to disagreements about the value of certain parameters, but rather are qualitative differences in the scaling properties of the models. The hazard ranges predicted by the models would be even less in mutual accord than is the case, if it were not for the fact that the slumping models are truncated before they can diverge too much. We thus identify lack of definite knowledge of the entrainment process during the slumping phase as a major contribution to the uncertainty in the hazards associated with heavy gas dispersion.

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8. APPENDIX A

Some examples

To give a more concrete idea of some of the models reviewed we append here a discussion of some specific cases.

8.1 Model parameters

Model	Edge entrainment	Top entrainment
Van Ulden (1974)	$\alpha = 0$	-
Germeles and Drake (1976)	-	$\alpha_{G.D.} = 0\cdot 1$
Fryer and Kaiser (1979)	$\alpha = 0\cdot 6$	$\alpha' = 0\cdot 15$
Picknett (1978)	$\alpha = 0\cdot 82$	$\beta_p = 0\cdot 15$
Fay and Ranck (1981)	-	$c_1 = 2.5$ $c_2 = 0\cdot 5$

The parameters for various models are given in the above table. Eidsvik's model has rather more parameters which take the values

$$\alpha = 0\cdot 5; \quad \alpha_4 = 3\cdot 5; \quad \alpha_6^* = 11\cdot 67; \quad \alpha_3 = 1\cdot 3$$

and c_f is given for different roughness lengths as

z_0	2 mm	10-20 mm	150 mm
c_f	$7\ 10^{-3}$	$1\cdot 4\ 10^{-2}$	$5\ 10^{-2}$

We take K = 1 for all the models to give a common starting point.

8.2 Dispersion in calm conditions

The distinction between atmospheric turbulence models and mechanical turbulence models is clearest in calm conditions where there is no atmospheric turbulence.

Thus the models of Picknett, Fryer and Kaiser, and of Fay and Ranck have no top entrainment in this case and reduce to the same behaviour as in Van Ulden's model. The models of Germeles and Drake and of Eidsvik, on the other hand, give an increasing height function as we have discussed in Section 4.2. The slumping behaviour is taken to continue indefinitely in all the models. If the safety criterion is taken to be $\Delta' < \Delta_c'$ then the hazard range L is given by

$$\frac{L}{R_0} = \left(\frac{\Delta_0'}{\Delta_c'}\right)^{1/2\alpha} \qquad \text{... (8.2.1)}$$

in the "ambient turbulence" models and by

$$\frac{L}{R_0} = \left\{\frac{\left[\frac{\Delta_o'}{\Delta_c'}\right] + 0\cdot 022\left[\frac{R_o}{h_o}\right] - 1}{0\cdot 022\left[\frac{R_o}{h_o}\right]}\right\}^{1/3} \qquad \text{... (8.2.2)}$$

in the model of Germeles and Drake. Equation (8.2.2) becomes arbitrarily large for large h_0/R_o. On the other hand the hazard range (at ground level) becomes technically infinite from (8.2.1) in the no entrainment limit ($\alpha \rightarrow o$). (This exposes the models' inadequacies in considering conditions near the ground rather than anything else). The model of Eidsvik is qualitatively similar to that of Germeles and Drake.

8.3 The experiment of Van Ulden (1974)[4]

To model this experiment we set $u_W = 3$ m/s, $u_* = 0 \cdot 25$ m/s with category D weather. The initial cloud parameters are $R_0 = 12$ m, $V_0 = 2400$ m^3 and $\Delta_0' = 0 \cdot 25$.

Van Ulden (1974) compares his model, in detail, with the data; we shall merely compare the models with each other. We shall follow the cloud, in each case, as far as the transition to passive behaviour.

The height and density of the cloud as a function of radius are given in Figures 2, 3, and the downwind motion is shown in Figure 4.

Some comments are in order. The lack of entrainment in Van Ulden's model gives a very dense low cloud some two hundred metres downwind when the transition to passive dispersions takes place. By contrast both Picknett and Fryer and Kaiser predict that the cloud has diluted by more than a factor of ten before the onset of passive dispersion. These will thus give rather lower hazard ranges. The model of Germeles and Drake in this case is almost entirely a passive one: the criterion $dR/dt = u_W$ used for the transition means that the slumping has barely begun before it is terminated in favour of a passive model. With reference to the previous discussion of Eidsvik's model we note that $\xi^2 = 0 \cdot 8$ and $\chi^2 = 4 \cdot 6$. This means that mechanical turbulence is never the dominant factor for this release, and that the asymptotic "passive" behaviour ($\hat{r} \gg \xi\chi$) sets in relatively early. (c.f. Germeles and Drake).

8.4 Scaling behaviour

In Figure 5 we give just one possible illustration of the scaling behaviour discussed in Section 5. We observe if R_0/h_0 and $\Delta_0' h_0$ are held constant while $R_0 h_0$ and Δ_0' are varied then all the models scale except those of Fryer and Kaiser and of Cox and Carpenter. (This is entirely due to the non-linear behaviour of Richardson number with height in these models). Figure 5, then, shows the variation of $\hat{h}$ ($\hat{r}$) for these models as one changes $R_0 h_0$ and Δ_0' by two orders of magnitude keeping the above combinations fixed. The relevant constant, γ differs (as $h_0^{0.52}$) by just over one order of magnitude between the two cases (which correspond roughly to wind tunnel dimensions and large field trials).

More generally one can vary h_0, R_0 and Δ_0' keeping two combinations fixed in such a way that different sets of models exhibit the scaling property. (For example, if h_0 and $R_0 \Delta_0'^{-3/2}$ are held fixed the models of Fryer and Kaiser, Cox and Carpenter, Picknett, and Van Ulden scale but the others do not).

9. APPENDIX B

Notation

(i) Subscripts and superscripts

a	refers to ambient air
g	refers to contaminant gas
o	refers to initial values (except t_0 and z_0)
^	refers to dimensionless variables

(ii) Common quantities

A	top area of cloud
b	negative buoyancy: $V \Delta' g/\pi$
c	mass fraction
c_p (c_{pa}, c_{pg})	specific heat
g	acceleration due to gravity
h	height of cloud
$\hat{h} = h/h_0$	dimensionless height variable
K	parameter in the front velocity
l	turbulence length scale
m (m_a, m_g)	mass of cloud (air, gas)
M_a, M_g	molecular weights
R	radius of cloud
$\hat{r} = R/R_0$	dimensionless radius variable
Ri	Richardson number
t	time
t_0	time scale
$\hat{t} = t/t_0$	dimensionless time variable
T (T_a)	temperature of cloud (air)
U_f	front velocity
U_E	edge entrainment velocity
U_T	top entrainment velocity
u_w	wind velocity
u_*	friction velocity
$\hat{u}_E$	non-dimensionalised edge entrainment velocity
$\hat{u}_T$	non-dimensionalised top entrainment velocity
V	Volume of cloud
$\hat{v} = V/Vo$	dimensionless volume variable
x	downwind distance
z	vertical co-ordinate
z_0	roughness length

α	edge entrainment coefficient
β	top entrainment parameter
γ	effective top entrainment parameter
Δ	$(\varrho - \varrho_a)/\varrho$
Δ'	$(\varrho - \varrho_a)/\varrho_a$
η	concentration
Λ	$1 - M_a/M_g$
ϱ (ϱ_a)	density of cloud (air)

(iii) Parameters of individual models

The letter given in brackets is the original authors designation where it differs from ours.

Van Ulden (1974)

K (c)	as above
α	as above

Germeles and Drake (1975)

K ($\sqrt{k}$)	as above
$\alpha_{G.D.}$ (α)	top entrainment coefficient

Fay (1980)

K (α)	as above
α (c_T/α)	as above

Fryer and Kaiser (1979)

h_r (30·17 m)	reference height in defining turbulent length scale
l (l_s)	as above
u_1	turbulence velocity
α (α_*)	as above
α'	top entrainment coefficient
μ (0·48)	parameter in turbulent length scale
σ_y	Gaussian plume width function

Picknett (1978)

α	as above
β_p (β)	top entrainment coefficient
K (γ)	as above

Eidsvik (1980)

c_f	"surface drag coefficient"
K (α_1)	as above
v_E (v)	turbulence velocity
α (α_5)	as above
α_3	turbulence velocity coefficient

α_4	top entrainment coefficient
α_6^*	density interface turbulence suppression parameter
ξ	initial front velocity/wind velocity parameter
χ	effective turbulence suppression parameter

Fay and Ranck (1982)

c_1 c_2	top entrainment parameters
K (α)	as above
ϕ	effective turbulence suppression parameter

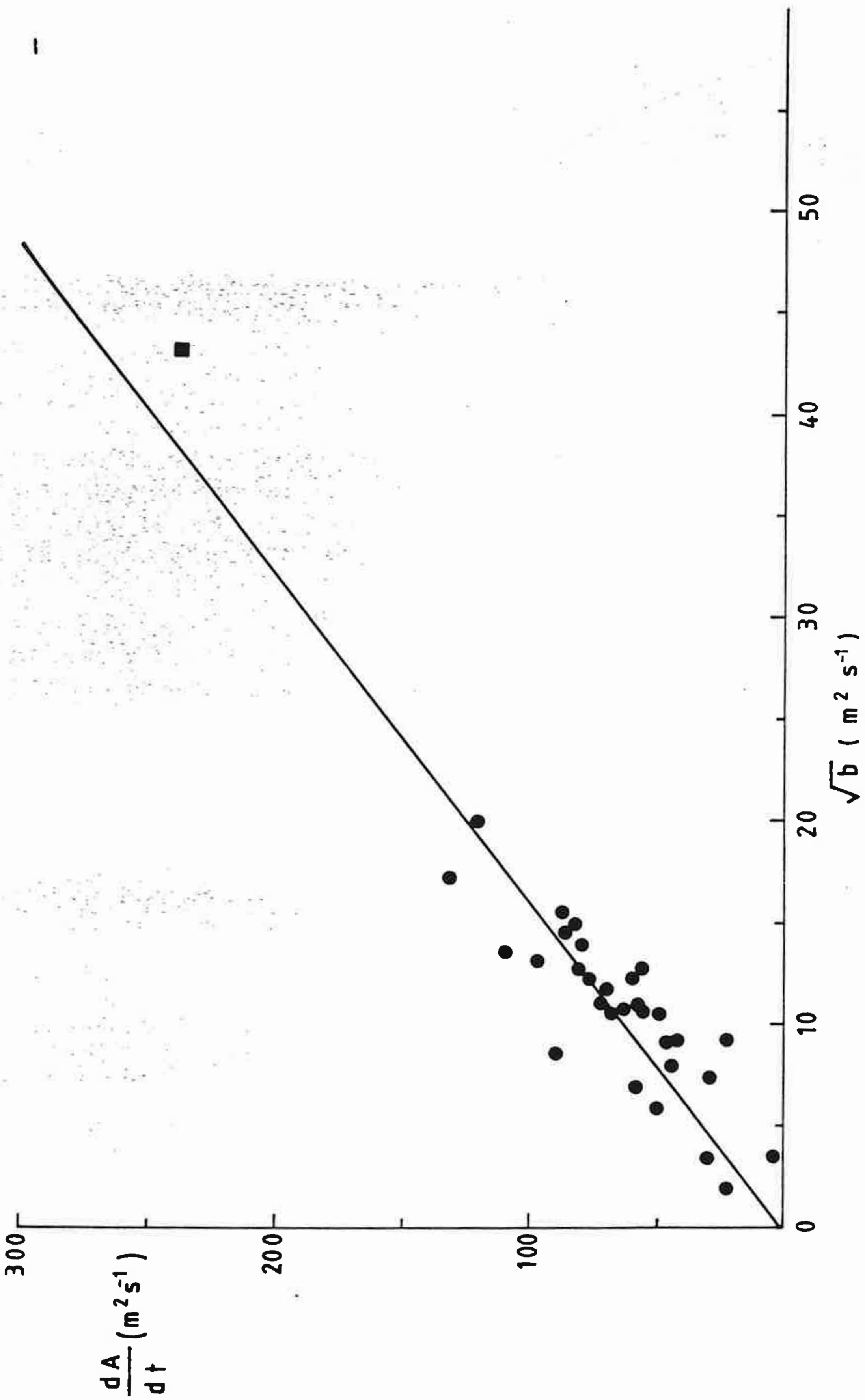

FIG. 1: The rate of change of the area of the cloud against the buoyancy variable. The K = 1 prediction is shown. The circles are the experiments of Picknett (1978); the square is that of Van Ulden (1974).

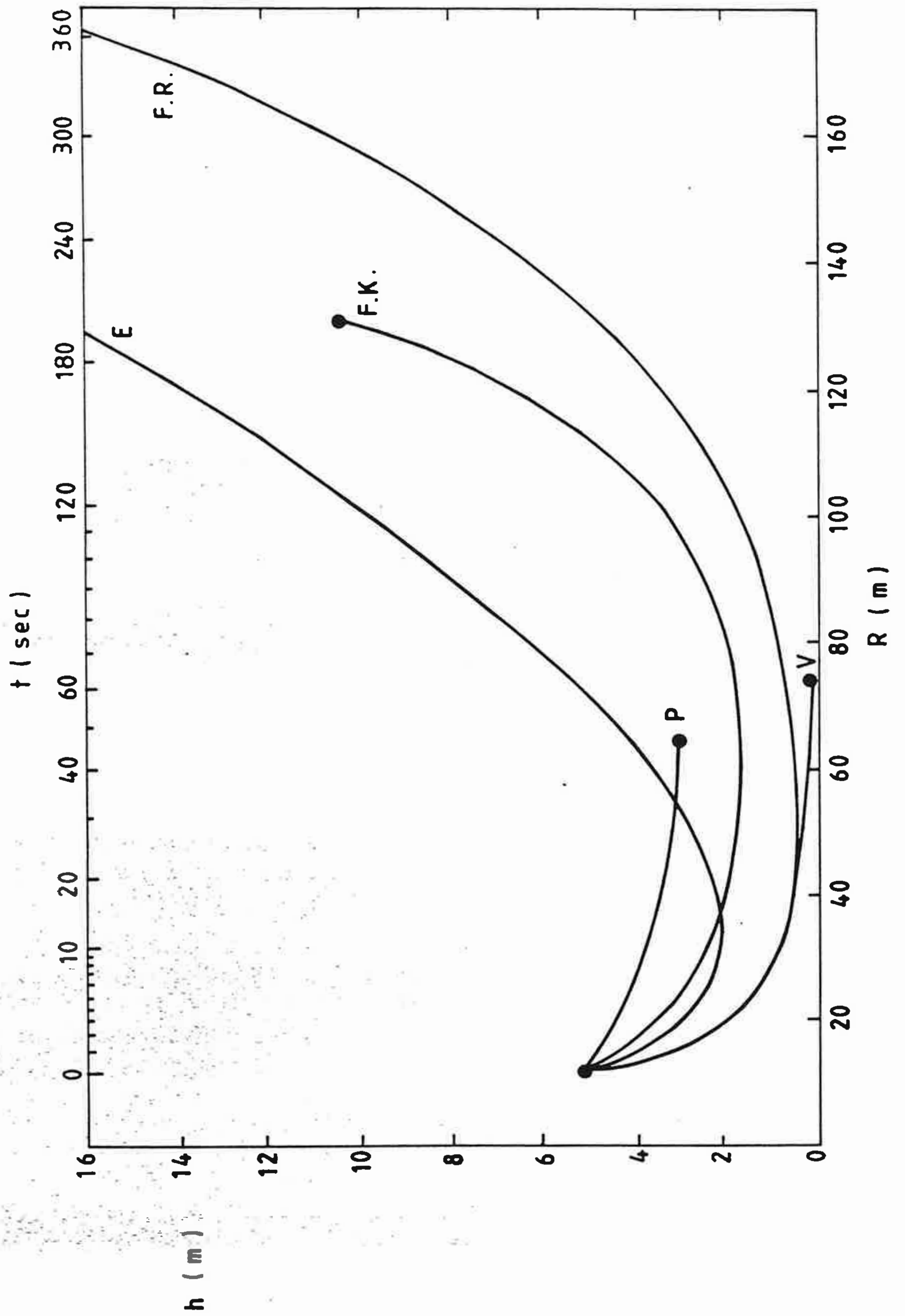

FIG. 2: **The height of the cloud as a function of radius (time) for the conditions described in Section A3, as predicted by the models of Van Ulden (V), Picknett (P), Fryer and Kaiser (F.K.), Eidsvik (E), and Fay and Ranck (F.R.). The models with a separate passive phase are followed up to the transition. The model of Germeles and Drake (not shown) goes passive at (R, h) = (14 m, 4·1 m).**

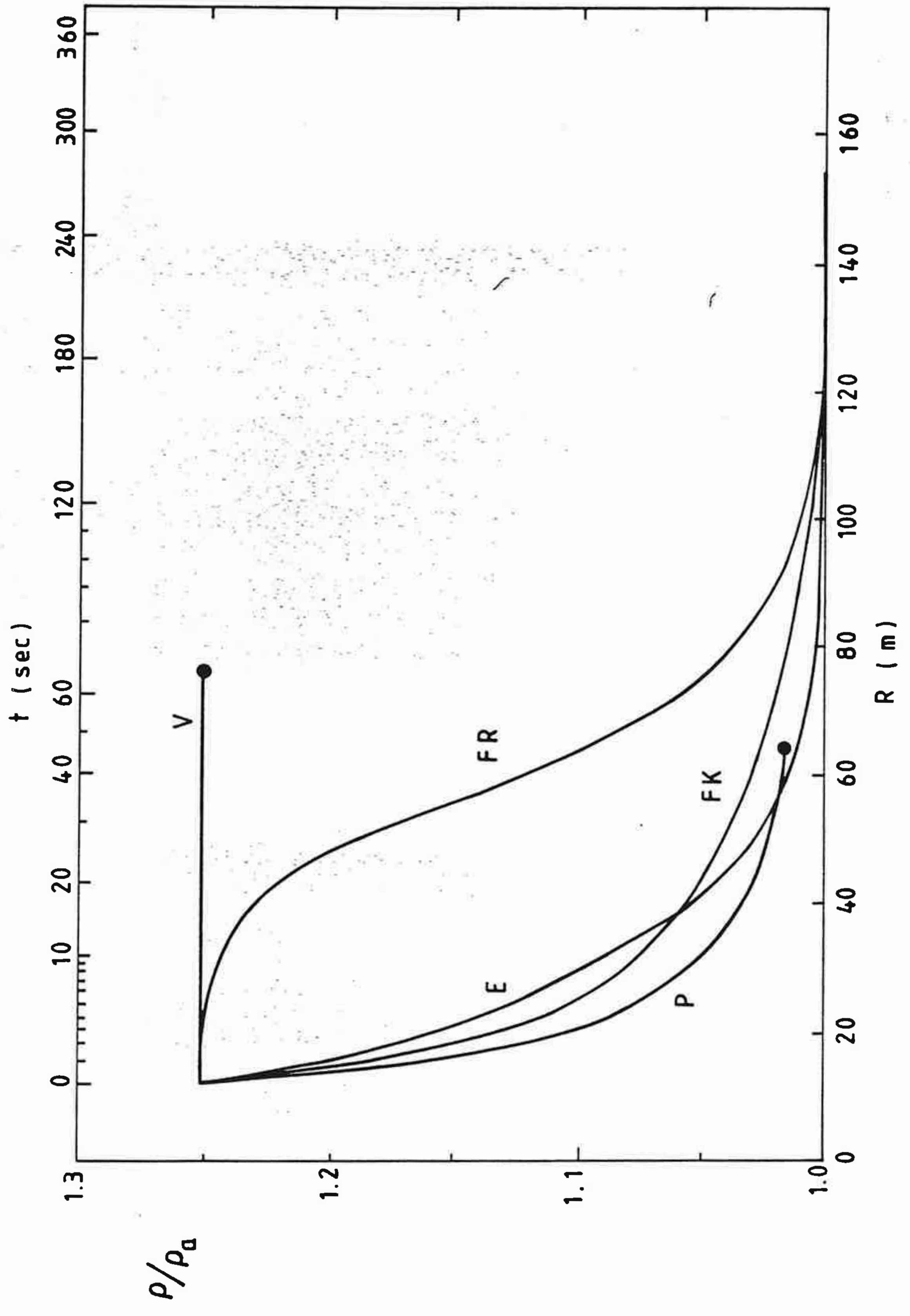

FIG. 3: The density of the cloud as a function of radius (time) corresponding to the models and release conditions of Fig. 2. This, together with Fig. 2, highlights the effect of the different entrainment models during the first few minutes of expansion. (The model of Germeles and Drake (not shown) goes passive with (R, ϱ) = (14 m, $1 \cdot 24$ ϱ_a)).

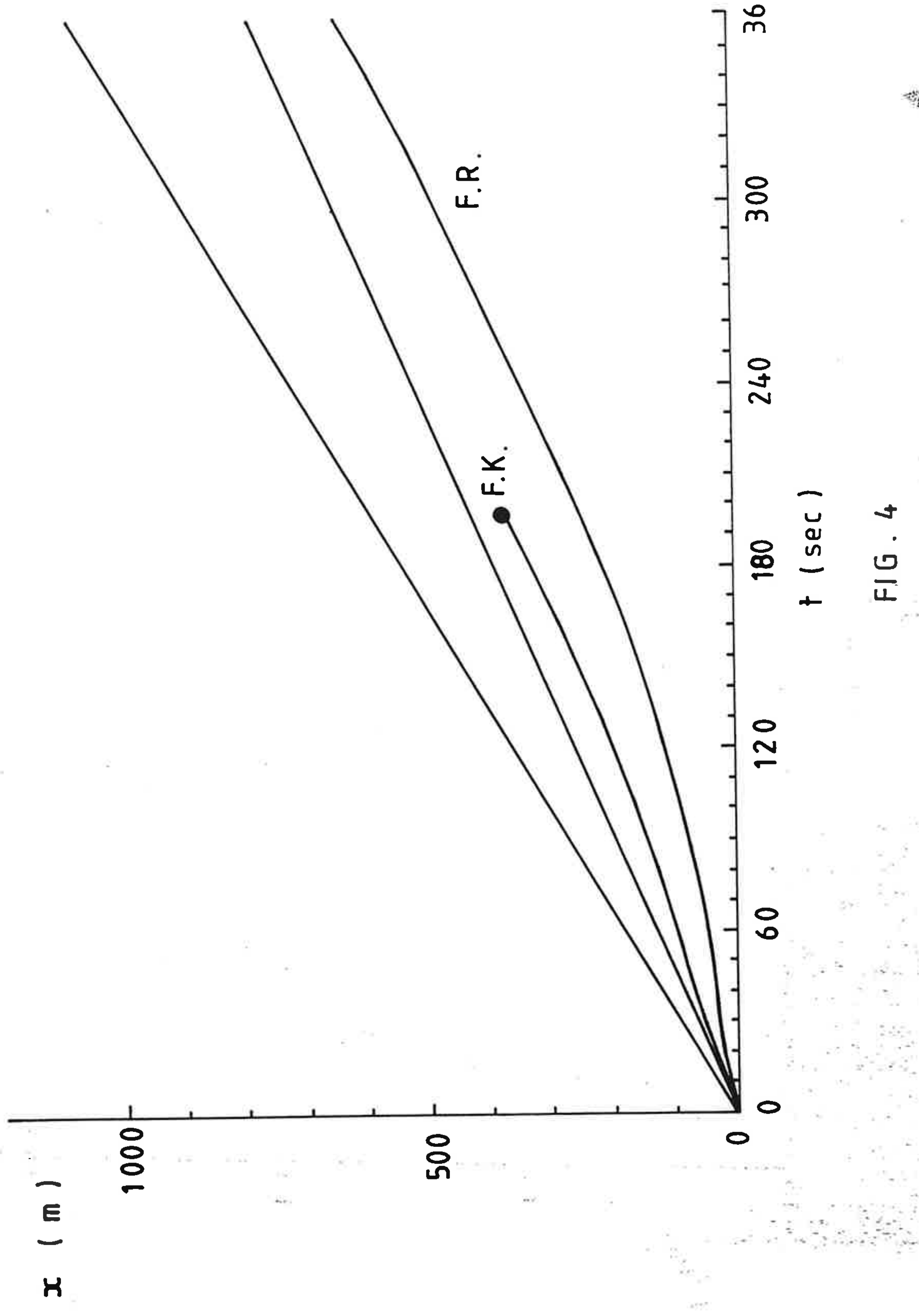

FIG. 4: **The downwind distance of the cloud illustrated in Figs 2, 3 as a function of time. The upper and lower straight lines correspond to assuming a constant velocity given by the wind speed at a height of 10 m and at half the initial height of the cloud respectively. (A logarithmic wind profile is assumed). Also shown are the predictions of the model of Fryer and Kaiser where the velocity is u_w (0·5 h) and of that of Fay and Ranck where the velocity is u_w (0·4 h). The effect of the slumping on the velocity is clearly seen, especially in the model of Fay and Ranck where the cloud becomes very low for a considerable period.**

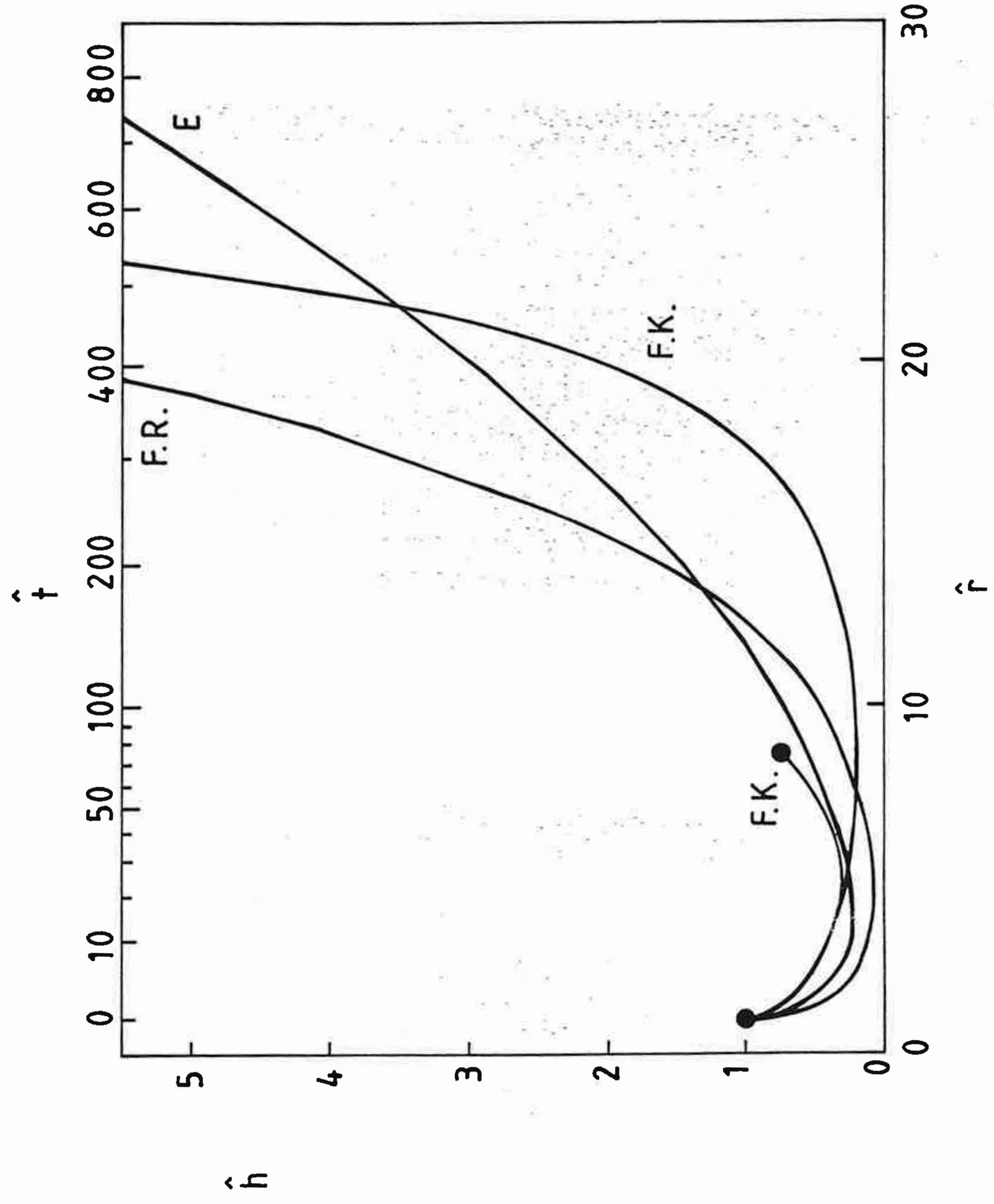

FIG. 5: The function $\hat{h}$ ($\hat{r}$) for releases with (R_0, h_0, ϱ_0) = (7 m, 13 m, 1·05 ϱ_a) and (7 cm, 13 cm, 6 ϱ_a). The combinations (R_0/h_0) and (h_0 Δ_0') are each the same for the two releases. Accordingly the only models which do not scale are those of Cox and Carpenter and of Fryer and Kaiser (F.K.) for which the two curves are shown. The models of Eidsvik (E) and of Fay and Ranck (F.R.) are shown for comparison. (Because of the large density difference the transition to passive behaviour in the model of Fryer and Kaiser occurs at very different $\hat{t}$.)

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